Digital Media Mergers: Theory and Application to Facebook-Instagram

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Abstract

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We present a new model of competition and mergers between two-sided media platforms with targeted advertising. The model includes familiar forces describing how platforms set ad load given user-side and advertiser-side substitution patterns, but adds new insights around how how user overlap and preference heterogeneity determine how competing platforms set ad load. We apply the model to evaluate the proposed separation of Facebook and Instagram. Using new data and new analyses of earlier randomized experiments with Facebook and Instagram users, we provide model-free evidence on user overlap, diversion ratios, price elasticity, and other parameters, and then estimate a structural model of this two sided market. Preliminary counterfactual simulations suggest that separating Facebook and Instagram would transfer significant surplus from platforms to advertisers, impose a small welfare cost on users, and decrease total welfare by a small amount. The total welfare gain would be much larger if separated platforms could avoid inefficient ad duplication across multi-homing users.

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It is hard to overstate the importance of quality and price for users and advertisers in digital markets. According to one estimate, there are now 5 billion internet users worldwide, averaging 6.6 hours online each day (Kemp 2023). More than half of the $680 billion yearly global advertising budget is spent online (Dentsu 2022). Given this importance, there is significant concern about digital platforms' market power and how this might affect outcomes for users and advertisers (Stigler Center 2019; CMA 2020; Scott Morton and Dinielli 2020).

Facebook and Instagram are a key example of this debate. In 2020, the U.S. Federal Trade Commission (FTC) sued Meta for antitrust violations, proposing that Instagram and WhatsApp be divested to restore competition for the benefit of both users and advertisers. The FTC and Meta disagreed over market definition, the extent of Meta's market power, and how divesting Instagram might affect user experiences, advertising prices, and other outcomes. The FTC argued that Facebook and Instagram have very high market share in the "personal social networking" market, which includes Snapchat but excludes TikTok or other communication or entertainment services. Meta (2021) counterargued that "The FTC's fictional market ignores the competitive reality: Facebook competes vigorously with TikTok, iMessage, Twitter, Snapchat, LinkedIn, YouTube, and countless others to help people share, connect, communicate or simply be entertained."

In this paper, we consider an important subset of these questions. In theory, how do mergers (or separations) of media platforms affect advertising loads and total surplus? Empirically, how do users substitute between Facebook, Instagram, and other apps? Quantitatively, how might ad loads and prices change if Facebook and Instagram were separated, and how would those changes affect total surplus?

We begin with a model of social media as a two-sided market. While the model nicely applies to the Facebook-Instagram case, it is generally useful for understanding platform competition with targeted advertising. Heterogeneous users and advertisers continuously allocate their time and ad spending between two platforms (e.g., Facebook and Instagram) and a non-strategic outside option. The platforms set advertising load, accounting for how higher ad load decreases both user time-on-platform and equilibrium ad prices.

The model includes familiar forces from Rochet and Tirole (2003) and Anderson and Coate (2005). When the two platforms are managed jointly, the monopolist doesn't internalize how ads reduce user surplus and increase advertiser surplus, so profit-maximizing ad load could be either above or below the social optimum depending on users' ad elasticity and advertisers' demand elasticity. When the two platforms are separated, the ad loads can increase or decrease depending on user and advertiser diversion ratios.

Our model is novel in capturing targeted advertising. Ad markets clear at the user level, with user-specific prices. When the platforms are separated, there is inefficient duplication: if the separated platforms don't share data, when both users and advertisers multi-home, the platforms may inefficiently impress a given user with extra ads from a given advertiser. Such a model can easily become intractable, and prior work such as Athey, Calvano, and Gans (2018) has required strong assumptions and can deliver results such as discontinuous reaction functions and mixed strategy equilibria that may not reflect actual market conditions. A key insight is that two assumptions substantially simplify the model: (i) ad click-through rates are independent across advertisers, and (ii) click-through rates are uniformly distributed across consumers.

Under these assumptions, we derive closed form expressions for the effect of duplication and use them to characterize pure strategy equilibria with novel but intuitive comparative statics. Specifically, if separated platforms cannot avoid inefficient duplication, separating platforms increases ad load by more when there is more user overlap across platforms. This is because (i) overlap increases advertiser losses from duplication, which softens advertiser demand; and (ii) there is a "business stealing" strategic incentive to reduce marginal overlap by expanding ad load to additional users whom the other platform is less likely to impress.

The model highlights that merging or separating digital platforms such as Facebook and Instagram has a theoretically ambiguous effect: it could either increase or decrease total surplus. However, these effects depend on a specific set of observable empirical statistics: user responses to higher prices and ad loads, the user diversion ratio between the two platforms, the extent of multi-homing and multi-homers' joint distribution of preferences to use the two platforms, as well as advertisers' price response.

We then apply the model to predict how separating Facebook and Instagram would affect those platforms' incentives to set ad load, and the effects that would have on total surplus. In the empirical part of the paper, we first provide descriptive empirical evidence on these parameters. We report results from the 2020 Facebook and Instagram Election Study ("FIES") implemented by Allcott et al. (2024). FIES includes two randomized experiments with nationally representative samples of 23,415 Facebook users and 21,249 Instagram users, respectively, in which randomly selected treatment groups were paid to deactivate Facebook or Instagram for six weeks. If the FTC's market definition were correct, users would primarily substitute between Facebook, Instagram, and Snapchat. In reality, there only very limited diversion between Facebook and Instagram, implying that (at least in the short run), joint ownership does not generate much additional market power on the user side.

We then analyze data from the Digital Addiction ("DA") experiment implemented by Allcott, Gentzkow, and Song (2022). DA is a randomized experiment with 2,053 Facebook and/or Instagram users in which a randomly selected treatment group was paid to reduce time spent on (but not necessarily deactivate) Facebook, Instagram, and other social media apps for three weeks. In the DA data, only about 12 percent of Instagram users are single-homers (i.e., don't use Facebook), while 32 percent of Facebook users are single-homers (i.e., don't use Instagram). This implies that there will be an asymmetry in the two platforms' incentives to increase ad load in the model. Among multi-homers, there is a wide dispersion in time-on-platform, which dampens platforms' business-stealing incentive in the model.

We import the Goli et al. (2018) estimate of the elasticity of user time-on-platform with respect to ad load, which suggests that users' time-on-platform is very inelastic to ads. The DA data suggest that users are very price elastic: 41–50 percent of Facebook and Instagram use is worth less than \$2.50 per hour to users. These two results suggest that ad loads could change substantially in modeled counterfactual equilibria and that this would not significantly affect consumer surplus.

We use these empirical moments to estimate the model's structural parameters. We impose a quadratic functional form on users' utility from time on the two platforms, and we identify the ad disutility, price responses, platform substitutability, and distribution of utility intercepts from the observed ad and price elasticities, diversion ratios, and time use distribution. We identify advertisers' aggregate price elasticity from the platform's first-order condition, in the spirit of how Berry, Levinsohn, and Pakes (1995) identify marginal costs from a Nash-Bertrand pricing assumption. Intuitively, if Facebook and Instagram set lower ad load (and thus higher prices), this is rationalized by inferring more inelastic advertising demand.

We use the estimated model to simulate the effects of separating Facebook and Instagram. In our current model, separation increases ad load on both platforms, reducing prices and time use. Ad load increases by more on Instagram: since a larger share of users are multi-homers, Instagram faces stronger incentives to increase ad load relative to Facebook to reduce the chance that marginal impressions are duplicated. The potential increase in advertiser surplus from higher ad load is attenuated by inefficiencies due to wasted impressions. Total surplus falls by 0.9 percent, since higher advertiser surplus is offset by lower consumer and platform surplus. Combined advertiser and consumer surplus rises by 1.4 percent. Overall, separation mostly transfers surplus from platforms to advertisers.

There are several important caveats. Perhaps most importantly, we consider only one margin on which competition affects welfare: equilibrium ad loads and prices in an otherwise static market. The Federal Trade Commission (2020), Scott Morton and Dinielli (2020), and others point to other potential effects of separating Facebook and Instagram (or other large digital media platforms), including entry of new businesses that would attract customers through social media ads or entry of new competing media platforms. Moreover, we do not consider how competition between Facebook and Instagram might induce the two apps to improve quality or change data sharing and privacy practices, or how Meta can currently combine data from both Facebook and Instagram to improve ad targeting and attribution. If Instagram were separated without access to Facebook's targeting and attribution technologies, ad targeting would likely become much worse, harming both users and advertisers.

Second, even within the context of our model, our empirical calibrations are imperfect. For example, our user diversion ratios are identified from a sample that selected into the experiment during the 2020 election, and the diversion ratios might well be different over a period longer than six weeks. Our estimate of the advertiser price response from the platform's first-order condition requires structural assumptions such as constant and exogenous profits per click and no change in user overlap with other media platforms.

Our work builds on several important literatures. First, we extend theoretical literatures on platform competition in general (Rochet and Tirole 2003; Armstrong 2006; Rochet and Tirole 2006; Weyl 2010; Rysman 2009) and specifically in media markets (Anderson and Coate 2005; Chen 2024;

Bergemann and Bonatti 2011; Ambrus, Calvano, and Reisinger 2016; Anderson and De Palma 2012; Prat and Valletti 2022; Anderson and Peitz 2023). Second, we extend work studying the effects of multi-homing in advertising markets, including duplication and incremental pricing (Ambrus, Calvano, and Reisinger 2016; Zubanov 2021; Anderson, Foros, and Kind 2018; Athey, Calvano, and Gans 2018; Gentzkow et al. 2024; Prat and Valletti 2022; Anderson and Peitz 2023). Third, we build on the work studying media industry mergers and separations (Berry and Waldfogel 2001; Benzell and Collis 2022; Chandra and Collard-Wexler 2009; Fan 2013; Gentzkow, Shapiro, and Sinkinson 2014; Jeziorski 2014). Fourth, we extend prior work estimating diversion ratios from product availability experiments in social media (Collis and Eggers 2022; Allcott et al. 2020; Mosquera et al. 2020; Aridor 2022; Allcott et al. 2024) and other markets (Goldfarb 2006; Conlon and Mortimer 2013, 2021; Conlon, Mortimer, and Sarkis 2021).

While some of our empirical analyses use data from the FIES and DA experiments we previously implemented, we think of this paper as a material additional contribution studying different topics. The FIES papers were focused on the effects of Facebook and Instagram use on political outcomes around the 2020 election, while the DA paper was focused on estimating users' self-control problems and habit formation.

Sections 1–6, respectively, present the model, model-free empirical evidence, structural estimation, counterfactuals, and conclusion.

1 Model

1.1 Setup

There are two digital media platforms indexed by j. The platforms choose ad load α_j (in ads per unit time on platform) to maximize profits. Bold typeface indicates vectors—e.g., $\boldsymbol{\alpha}$ is the vector of ad loads on each platform. We assume that platforms have zero marginal cost, so (variable) profit equals ad revenue $R(\boldsymbol{\alpha})$.

There is a measure-N continuum of users indexed by i. Users choose time on each platform T_{ij} and numeraire consumption n_i to maximize utility $U_i(\boldsymbol{T}_i, n_i; \boldsymbol{\alpha})$. We assume that U_i is quasilinear in n_i, so changes in U_i correspond to changes in consumer surplus. We assume that users' utility or disutility from ad load accounts for expected consumer surplus from any purchases of advertised products, so we do not need to separately account for consumer surplus in advertisers' product markets.

There is a measure-A continuum of advertisers indexed by a. Each advertiser earns exogenous profit per ad click π_a. For example, π_a might equal a user's purchase probability (conditional on clicking on an ad) times the product's markup. The model is isomorphic if we redefine "clicks" as some other advertising result (such as impressions or purchases) or redefine "advertisers" as separate ad campaigns run by the same firm. Advertisers choose the quantity of clicks q_a to purchase from the platforms to maximize profits $\Pi_a(q_a; \pi_a)$.

The targeted advertising technology is as follows. An ad "campaign" involves m impressions of advertiser a's ads to each targeted user, where m is the platform's prediction of the optimal number of impressions for user i.[1] Define ω_{ia} as platform's prediction of user i's probability of clicking on a given ad impression during advertiser a's first ad campaign, which we call the "click-through rate." The click-through rate ω_{ia} is the same on both platforms: ads are equally effective if seen on either platform, and both platforms have the same targeting technology (even if separated). The click-through rate after the first campaign is $(1-\zeta_i)\cdot\omega_{ia}$, where ζ_i is the percent decrease in click-through rate on impressions from the first to the second campaign. This captures the diminishing returns to additional impressions.

As described below, there is a separate ad market for each user, with equilibrium price per impression p_i. The predicted cost per click is thus p_i/ω_{ia}, and the predicted profit per impression is thus $\omega_{ia}\pi_a$ and $(1-\zeta_i)\cdot\omega_{ia}\pi_a$ for impressions in the first and second campaign, respectively. For user i, define $H_i(x)\in[0,1]$ as the cumulative density function (CDF) of $\omega_{ia}\pi_a$ across advertisers.

The platform's contract offer to advertisers is to serve ads to the $\mathcal{U}_a(q)$ users with the lowest cost per click and charge total price $C_a(q)=\sum_{i\in\mathcal{U}_a(q)} mp_i$. Advertisers know $C_a(q)$ when choosing q.

We impose three assumptions that substantially simplify the analysis.

Assumption 1. Independent click-through rates: *$\omega_{ia}\perp\omega_{ia'}$, $\forall(a,a')$; $\omega_{ia}\perp T_{ij},\forall(a,j)$; and $\omega_{ia}\perp\zeta_i,\forall a$.*

Assumption 2. Identical uniform profits per impression: *$H_i(x)=\eta^{-1}x-\eta_0$, $\forall i$.*

Assumption 1 states that click-through rates are independent across advertisers and independent of time-on-platform. This rules out the possibility that some users are more or less valuable on average across advertisers. This assumption could be weakened by modeling distinct user types, such as high- or low-income people. Assumptions 1 and 2 together imply that a given change in $\boldsymbol{\alpha}$ has the same effect on all $C_a(q)$, and Assumption 2 facilitates straightforward demand aggregation across individual users.

We impose the regularity condition that $1-\zeta_i\leq p_i^m/(\eta\cdot(1+\eta_0))$, where p_i^m is the merged equilibrium price where platforms are constrained to serve at most one campaign to user i from advertiser a. The regularity condition implies that predicted profit per impression for the second campaign is weakly below the market clearing price were the platform to never show duplicated impressions, since $\eta\cdot(1+\eta_0)$ is the maximum profit per impression given Assumption 2. Thus, in the merged equilibrium, the platform never shows multiple campaigns from the same advertiser to a user, since it is weakly more profitable to show the first campaign from the marginal advertiser.

[1] Assuming constant impressions is isomorphic to assuming heterogeneous impressions m_i with $m_i\perp\omega_{ia}\pi_a$, $\forall i,a$ with $m\equiv E_i[m_i]$. We also show in Appendix A.1 that equilibrium prices are identical under heterogeneous $m_i\perp\omega_{ia}\pi_a$ and an alternative assumption where m_i is increasing in profits per impression and decreasing in prices, with a specific functional form.

1.2 Profits and Market Clearing Conditions

We now derive platform revenue as a function of ad load in two equilibria: (i) when the two platforms are merged, and (ii) when the two platforms are separated. In the merged equilibrium, the firm maximizes the sum of profits across the two platforms. Due to the assumption that $1 - \zeta_i \leq p_i^m / (\eta \cdot (1 + \eta_0))$, the firm does not duplicate a given ad campaign a to a given user i on both platforms. In the separated equilibrium, the platforms independently maximize profits and cannot coordinate to avoid duplicating impressions of the same campaign to a given user.

1.2.1 Merged Equilibrium

Advertisers choose the quantity of clicks to purchase from the merged firm. Advertiser profits equal product market profits net of advertising costs:

$$\Pi_a^m (q) = \pi_a \cdot q - C_a (q) . \tag{1}$$

Maximizing profits gives

$$\pi_a = C_a' (q_a) . \tag{2}$$

In words, advertisers purchase ads until the marginal cost per click equals profit per click.

Ad markets clear at the user level:

$$\overbrace{\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})}^{\text{supply}} = \overbrace{\sum_a m \cdot \mathbf{1} \left[i \in \mathcal{U}_a (q_a) \right]}^{\text{demand}} = \sum_a m \cdot \mathbf{1} \left[p_i \leq \pi_a \omega_{ia} \right] = Am \cdot (1 - H_i (p_i)) . \tag{3}$$

Rearranging equation (3) gives equilibrium price

$$p_i = H_i^{-1} \left(1 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})}{Am} \right), \tag{4}$$

where H_i^{-1} is the inverse CDF. We assume that η and η_0 are such that we always have an interior equilibrium. This equation shows that equilibrium prices are increasing in ad demand (the number of advertisers A and campaign size m) and decreasing in ad supply (ad load $\boldsymbol{\alpha}$ and time on platform $\boldsymbol{T}_i$).

The merged platform revenue is

$$R^m(\boldsymbol{\alpha}) = \sum_i \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot p_i . \tag{5}$$

1.2.2 Separated Equilibrium

In the separated equilibrium, advertisers choose the number of clicks q_j to purchase from each of the two platforms at total price $C_{aj} (q_j)$. Define $O_a (q_1, q_2)$ as the "overlap" function: the number of duplicated clicks purchased, which is weakly increasing in the number of clicks purchased on each platform. Advertiser profits account for that overlap:

$$\Pi_a^s(q_1, q_2) = \pi_a \cdot (q_1 + q_2 - O_a) + \underline{\pi}(q_1, q_2; O_a) - C_{a1}(q_1) - C_{a2}(q_2). \tag{6}$$

where $\underline{\pi}$ is the profits from duplicated impressions, given the overlap function. Define O'_{aj} as the derivative of O_a with respect to q_j. Maximizing profits gives

$$\pi_a \cdot \left(1 - O'_{aj}\right) + \underline{\pi}'_{aj}(q_{a1}, q_{a2}; O_a) = C'_{aj}(q_{aj}). \tag{7}$$

In words, advertisers purchase ads until the overlap-adjusted marginal cost per click equals profit per click. The first term reflects how an increase in q_j impacts profits from non-duplicated impressions. The second term equals the derivative of $\underline{\pi}$ with respect to q_j. In equilibrium, $\underline{\pi}'_{aj} = (1-\zeta_j)\cdot\pi_a\cdot O'_{aj}$, where ζ_j is the time-use weighed average of ζ_i on platform j for multi-homing users with positive time use on both platforms, and $\pi_a \cdot (1 - \zeta_j)$ is the marginal profit per *purchased* click for a user i', who is the marginal user impressed by advertiser a on platform j.[2] The left-hand side of equation (7) can therefore be written as $\pi_a \cdot \left(1 - \zeta_j O'_{aj}\right)$. Note that since ζ_j is a time-use weighted average of ζ_i among multi-homers, it is a function of $\boldsymbol{\alpha}$.

The user-level market clearing condition is analogous to the merged case, except that markets now clear separately on each platform j:

$$\overbrace{\alpha_j \cdot T_{ij}(\boldsymbol{\alpha})}^{\text{supply}} = \overbrace{\sum_a m \cdot \mathbf{1}\left[p_{ij} \leq \pi_a \omega_{ia} \cdot \left(1 - \zeta_j \cdot O'_{aj}\right)\right]}^{\text{demand}} = Am \cdot \left(1 - H_i\left(\frac{p_{ij}}{1 - \zeta_j \cdot O'_{aj}}\right)\right) \tag{8}$$

Rearranging equation (3) gives equilibrium price

$$p_{ij} = H_i^{-1}\left(1 - \frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{Am}\right) \cdot \left(1 - \zeta_j \cdot O'_{aj}\right). \tag{9}$$

Each platform's revenue is

$$R_j^s(\alpha_j; \alpha_{-j}) = \sum_i \overbrace{\alpha_j \cdot T_{ij}(\boldsymbol{\alpha})}^{\text{ad supply}} \cdot p_{ij}. \tag{10}$$

There are two sources of strategic interaction between platforms in the separated equilibrium. First, platforms compete on the user side due to time use complementarity or substitutability. Second, on the advertiser side, platform revenue is decreasing in O'_{aj}, the expected increase in duplicated clicks from an additional click purchased on j. The equilibrium value of O'_{aj} depends

[2] Since the separated platform believes the click-through rate for duplicated users to be ω_{ia}, the purchase of a click for a duplicated user involves ω_{ia}^{-1} impressions. Since the profit per impression for duplicated users is $\underline{\omega}_{ia}\pi_a$, the profit per purchased click is $\underline{\omega}_{ia}\pi_a/\omega_{ia} = \pi_a \cdot (1 - \zeta_i)$. Taking the conditional expectation for users with overlap gives the expression in the text.

on the rival platform's ad load.

Suppose that Assumption 1 holds and advertiser demand (q_{a1}, q_{a2}) satisfies the first-order conditions in equation (7). Let $\mathcal{U}_j$ be the set of users on platform j, and $N_j = |\mathcal{U}_j|$ indicate their number. We show in Appendix A.2 that the marginal overlap function evaluated at equilibrium ad demand is

$$O'_{aj}(q_{a1}, q_{a2}) = \frac{\sum_{i\in\mathcal{U}_j} \mathbf{1}[\alpha_j T_{ij}(\boldsymbol{\alpha}) \leq \alpha_{-j} T_{i,-j}(\boldsymbol{\alpha})]}{N_j} \in [0,1] \tag{11}$$

Equation (11) shows that marginal overlap depends on two dimensions of time use. First, marginal overlap is increasing in the fraction of users on platform j who multi-home (spend time on both platforms). If user i is a single-homer on i, then $T_{i,-j}(\boldsymbol{\alpha}) = 0$, and the indicator in the numerator is zero for that user.

Second, marginal overlap is decreasing as ad load increases relative to platform $-j$. To understand this point, consider two extremes. Suppose that for all multi-homers, $T_{ij} > T_{i,-j}$, and $\alpha_j = \alpha_{-j}$. Market clearing prices must be lower on platform j for all i, implying that the marginal click-through rate ω_{ia} is lower as well. This implies the marginal user impressed by an increase in quantity on platform j will not be served on platform $-j$, so no clicks are wasted. Suppose instead the opposite: $T_{ij} < T_{i,-j}$ for all i who multi-home. Market clearing prices will be higher on platform j for all i. The marginal user impressed on j will already be served on $-j$, so all additional clicks purchased are wasted.

Put differently, higher ad load on a platform decreases ad prices, allowing advertisers to expand their reach beyond the set of users that are more likely to also be impressed on a competing platform. Advertisers recognize that the marginal user impressed on a platform with greater reach is less likely to be impressed elsewhere. This increases demand.

1.3 Special Case: Homogeneous Users, Constant Click-Through Rate, and No Duplication

While specialized to digital media markets, our model includes familiar economic forces to existing models with two-sided platforms engaged in quantity setting games, such as Anderson and Coate (2005). To illustrate these forces, Appendix A.3 examines the social optimum, merged, and separated competitive equilibrium in a special case where identical users ($U_i = U, \boldsymbol{T}_i = \boldsymbol{T}, n_i = n$) have the same click-through rate on all ads ($\omega_{ia} = \omega_a$). We assume in the separated equilibrium, the two platforms coordinate to avoid the duplication effect, and that ζ_j is sufficiently high so that neither platform serves more than one campaign from an advertiser to a user.

The special case shows how the two-sided nature of media market platforms complicates standard antitrust welfare analysis. First, ad load in the merged competitive equilibrium may be higher or lower than the social optimum. The key disortion in the merged equilibrium is that the user time use is not directly priced, and hence enters nowhere in the platform problem. The socially optimal ad load equates the marginal welfare gain from advertisers with the marginal welfare loss for consumers. By contrast, the merged platform equates the increase in marginal revenue from higher ad load with the infra-marginal loss in revenue from lower prices. While the increase in marginal revenue equals advertiser surplus, the infra-marginal loss in revenue from lower prices may be higher or lower than marginal consumer surplus.

Second, ad load in the separated equilibrium may be higher or lower than in the merged equilibrium. Because merged equilibrium welfare may be higher or lower than is socially optimal, separation has an ambiguous effect on welfare. The effect of separation on ad load depends on user-side diversion. In particular, separated equilibrium ad load tends to be lower when platforms are substitutes, i.e. $\frac{\partial T_{-j}}{\partial \alpha_j} > 0$. This is because separation causes platforms to more aggressively compete on the user side, which restrains their increase in ad load.

Therefore, user diversion $\frac{\partial T_{-j}}{\partial \alpha_j}$ is a potentially important driver of the welfare effects of separation due to two-sided forces in our model. Our empirical approach places emphasis on credibly estimating this parameter.

1.4 Advertiser Side in Isolation

We now focus on the advertiser side of the market, assuming that time on platform $\boldsymbol{T}_i$ is exogenous but not necessarily homogeneous. We highlight two features of the model. First, rather than generic diversion ratios, merged and separated equilibrium ad load depends on novel user overlap statistics. Second, separating platforms impacts ad load both by changing strategic incentives and also the available advertising technology. For clarity, the results in this subsection assume $\zeta_j = 1$, so that duplicated impressions are fully wasted. See Appendix A.5 for derivations. We first discuss general results before illustrating them using numerical examples in Section 1.4.3.

1.4.1 Merged Equilibrium

The merged platform chooses $\boldsymbol{\alpha}^{e,m}$ to maximize revenue given in Equation (5). The solution is

$$\alpha_j^{e,m} = -\frac{\sum_i \alpha_{-j} \cdot T_{i,-j} \cdot \frac{\partial p_i}{\partial \alpha_j} + T_{ij} \cdot p_i}{\sum_i T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}}, \; j = 1, 2. \tag{12}$$

$$\text{where } \frac{\partial p_i}{\partial \alpha_j} = -\left(Am \cdot h_i(p_i(\boldsymbol{\alpha}))\right)^{-1} \cdot T_{ij} \text{ and } h_i = H_i'.$$

Monopolist ad load equalizes the marginal increase in revenue due to a direct increase in ad load and the marginal revenue decrease on infra-marginal impressions on both platforms. For intuition, suppose that all users are single-homers and have homogeneous time use if positive, so that $T_{ij} \in \{0, T_j\}$ and $\min(T_{i1}, T_{i2}) = 0 \; \forall i$. Applying Assumptions 1 and 2, equation (46) simplifies to $\alpha_j T_j = \frac{1}{2} Am \cdot (1 + \eta_0)$, the standard solution for a monopolist maximizing against a linear demand curve. In that case, the platform sets ad load independently on each platform.

Two forces shape ad load in our model relative to the standard monopolist incentive to restrict supply to increase price. First, the monopolist internalizes the effect of choice of ad load on platform

α_j on revenue from ads shown on platform $-j$. In particular, greater overlap implies lower ad load. Second, the monopolist considers the variance of time use across users. Since prices are set at the user level but ad load is not, time use variance impacts how revenue changes when ad load shifts.

1.4.2 Separated Equilibrium

In the separated equilibrium, platforms solve:

$$\alpha_j^{e,s} = \arg\max_{\alpha_j} \sum_{i \in \mathcal{U}_j} \alpha_j \cdot T_{ij} \cdot p_{ij} \tag{13}$$

Equilibrium ad load is therefore:

$$\alpha_j^{e,s} = -\frac{\sum_{i\in\mathcal{U}_j} T_{ij} \cdot p_{ij}}{\sum_{i\in\mathcal{U}_j} T_{ij} \cdot \frac{\partial p_{ij}}{\partial \alpha_j}},\ j = 1, 2. \tag{14}$$

First, suppose that the platforms can coordinate to avoid duplication. In this case, $p_{ij} = p_i$, given by equation (4), and $\frac{\partial p_i}{\partial \alpha_j} < 0$ is as in equation (12). This coincides with the merged platform solution if all users single-home. As the fraction of multi-homers rises, equation (12) indicates that merged equilibrium ad load rises relative to equation (14). Hence, separation reduces ad load by more as overlap increases. Intuitively, overlap increases the Cournot externality imposed by platform j increasing ad load on platform $-j$, which increases ad load in the separated equilibrium relative to the merged equilibrium.

Next, suppose platforms cannot coordinate to avoid duplication. In this case, p_{ij} is given by equation (9). Furthermore,

$$\frac{\partial p_{ij}}{\partial \alpha_j} = -\left(1 - O'_{aj}(\boldsymbol{\alpha}^{e,s})\right) \cdot (Am \cdot h_i(p_{ij}))^{-1} \cdot T_{ij} - \frac{p_{ij}}{\left(1 - O'_{aj}(\boldsymbol{\alpha}^{e,s}\right)} \frac{\partial O'_{aj}(\boldsymbol{\alpha}^{e,s})}{\partial \alpha_j}. \tag{15}$$

Without an integrated ad market, there is no Cournot externality mediated by overlap. However, the loss from duplication plays a similar role, where greater user overlap implies a greater increase in ad load in the separated equilibrium. For intuition, suppose that all users were single-homers, so O'_{aj} and its first derivative equal zero. Ad load would coincide with the merged equilibrium in equation (12). Positive overlap reduces ad load in the merged equilibrium and increases ad load in the separated equilibrium. The latter is because overlap reduces revenue from infra-marginal impressions (reflected in the first term in $\frac{\partial p_{ij}}{\partial \alpha_j}$) and because platforms have a strategic incentive to increase ad load to reduce overlap (reflected in $\frac{\partial O'_{aj}}{\partial \alpha_j}$ in the second term in $\frac{\partial p_{ij}}{\partial \alpha_j}$). We call the first term the "inframarginal effect," and we call the second term the "business stealing effect," where $\frac{\partial O'_{aj}}{\partial \alpha_j} \leq 0$ from the definition of O'_{aj} in equation (11).

Business stealing occurs because platforms with higher ad load have lower prices, which signals to advertisers that marginal clicks purchased are shown to users with lower CTRs. Since CTRs are user-specific across platforms, the marginal user impressed on the low-price platform is *not* impressed on the high-priced platform. Therefore, duplication from overlap reduces ad demand by *less* on the platform with higher ad load, all else equal.

The business stealing effect is also different from a standard Cournot externality because it can generate strategic complementarities across platforms, where an increase in ad load on j encourages $-j$ to increase ad load as well. When a rival's ad load increases, demand falls because marginal impressions are more likely to be duplicates. This increases ad load through the direct effect and strategic channels. By contrast, in our model ad load choices with no duplication are always strategic substitutes. We formalize this comparison in Proposition 1 in Appendix A.5, and illustrate it numerically in the next section.

Lastly, the possibility of duplicated impressions means the separated equilibrium has a social inefficiency relative to the merged equilibrium.

1.4.3 Numerical Examples

We explore how the above forces impact ad load in the separated relative to combined equilibrium through several numerical examples. We parameterize the distribution of time use with a measure O of multi-homers with $\boldsymbol{T}_{mi} \sim \mathcal{N}\left(\frac{\mathbf{1}}{\mathbf{2}}, \boldsymbol{\Sigma}_T\right)$, and a measure N_j of single-homers $T_{si} = 1$. Let $\mu_j \equiv O/N_j$, the fraction of multi-homers. Unless otherwise specified, we set $\zeta_j = m = 1$ and the remaining model parameters to their estimated value from Section 4.

Example 1: Partial overlap (with no duplication). We first focus on how partial user overlap impacts the magnitude of the Cournot externality when separated platforms can coordinate to avoid duplication. We set $\boldsymbol{\Sigma}_T = 0$, so that multi-homers split one unit of time equally across platforms.

The merged equilibrium solution is standard linear Cournot: $\alpha_j^{e,m} = \frac{1}{2} Am \cdot (1 + \eta_0)$. In separated equilibrium, reaction functions are:

$$\alpha_j^{e,s,i}(\alpha_{-j}) = \frac{2 - \mu_j}{(4 - 3\mu_j)} Am \cdot (1 + \eta_0) - \frac{1}{2} \frac{\mu_j}{(4 - 3\mu_j)} \alpha_{-j} \tag{16}$$

When there is no overlap ($\mu_j = 0$), each separated platform behaves as a monopolist. As overlap increases, separated platforms respond more to their rival's actions, as they have a greater impact on revenue, and interalize less the price impact of increased ad load. Appendix Section A.5.1 derives a closed-form expression for separated equilibrium ad load, and shows that the percent change relative to the merged equilibrium *only* depends on overlap statistics and is independent of ad demand parameters.

Figure 1, panel (a) plots reaction functions and equilibria in to show these forces concretely. Facebook reaction functions to Instagram ad load are plotted horizontally, and Instagram reaction functions to Facebook ad load are plotted vertically. Equilibria are plotted wth black dots where the reaction functions intersect. Black lines plot vertical reaction functions when $\mu_j = 0$, because platforms ignore their rival's actions. The blue and orange solid lines plot reaction functions when $\mu_j = 1$ for both platforms, indicating a 40% increase in ad load relative to the monopolist equilibrium. The dashed lines plot reaction functions given empirically-observed μ_j, with $\mu_{IG} > \mu_{FB}$. Since Instagram has more overlap than Facebook, it will internalize less of the impact of increased ad load on equilibrium prices, and hence increase ad load by much more than Facebook.

Example 2: Duplication (with no partial overlap). We next consider a separated equilibrium where platforms cannot coordinate to avoid duplication. To focus on the role of duplication, we assume all users are multi-homers with heterogeneous time use parameterized by $\mathbf{\Sigma}_T = \text{diag}\left(\sigma^2, \sigma^2\right)$. Heterogeneous time use ensures that the marginal overlap function is continuous and differentiable.

Reaction functions no longer have a closed form, but are given by:

$$\alpha_j^{e,s,i}(\alpha_{-j}) = \arg\max_{\alpha_j} \left(1 - O'_{aj}\left(\alpha_j, \alpha_{-j}\right)\right) \cdot \eta \left(\sum_i \alpha_j T_{ij} \cdot \left(1 + \eta_0 - \frac{\alpha_j T_{ij}}{Am}\right)\right), \tag{17}$$
$$\text{where } O'_{aj}(\alpha_j, \alpha_{-j}) = \Pr(\alpha_j T_{ij} \leq \alpha_{-j} T_{i,-j}),$$

and where we substitute for p_{ij} in equation (13) and apply the no-overlap assumption. Rival ad load α_{-j} only enters the expression through marginal overlap O'_{aj}. This makes clear the inframarginal effect (shifting levels) and business stealing effect (shifting strategic incentives) generated by duplication. Moreover, the variance of time use, controlled by σ^2, will alter separated platform incentives. When σ^2 is very low, platforms can precisely affect marginal overlap and user prices through changes in ad load, amplifying strategic differences relative to the merged equilibrium. When σ^2 is very high, changes in ad load have little impact on marginal overlap, dampening differences relative to the merged equilibrium.

Figure 1, panel (b) illustrates these points. The solid blue and orange lines plot Facebook and Instagram reaction functions when σ^2 is relatively low. Ad load increases significantly. Because platforms have fine control over marginal overlap, the business stealing effect is strong, making choice of ad load a strategic complement. The dashed lines plot reaction functions when σ^2 is high. Ad load still shifts out due to the direct effect. However, platforms have much less control over marginal overlap through choice of ad load, severely dampening the business stealing effect.

Example 3: Duplication and partial overlap together. Finally, we consider the impact of duplication and partial overlap jointly. Reaction functions are the same as in equation (17), except that $O'_{aj}(\cdot) = \mu_j \Pr(\alpha_j T_{ij} \leq \alpha_{-j} T_{i,-j})$. This means the impact of duplication is attenuated if overlap μ_j is low on a given platform.

Figure 1, panel (c) plots the resulting reaction functions, where all reaction functions use the empirical overlap μ_j on Facebook and Instagram. First, partial overlap ($\mu_j < 1$) dampens the impact of duplication, since each platform has a user population for whom strategic incentives do not change relative to the merged equilibrium. Second, the business stealing motive appears stronger for Instagram than Facebook. This is because Instagram has greater overlap, magnifying the returns to business stealing. Finally, higher time use variance again dampens the business stealing effect, although to a lesser degree as it is already dampened by overlap.

Summary. In an environment with exogenous but heterogeneous time use, separating platforms increases ad load by more when user overlap across platforms is greater. If separated platforms can coordinate to avoid duplication, this is because user overlap increases the externality of lower prices imposed by a platform on their rival from an increase in ad load. If separated platforms cannot avoid duplication, this is because: (i) overlap increases anticipated advertiser losses from duplication, which softens advertiser demand, and (ii) a "business stealing" strategic incentive to expand ad load to the population not already impressed to reduce marginal overlap.

1.5 Full Equilibrium

We now consider the full equilibrium with endogenous and heterogeneous time use under different ownership structures. The formal analysis, presented in Appendix A.6, combines expressions in Sections 1.3 and 1.4. We maintain the assumption that $\zeta = 0$ throughout. We summarize the results below.

The social planner chooses ad load to equalize the marginal aggregate user welfare cost of ads and the marginal aggregate advertiser welfare benefit of ads. Relative to the planner problem in Section 1.3, the planner calculates aggregate costs and benefits by summing across heterogeneous individuals.

The merged platform increases ad load on each platform to the point that the revenue gain from increased aggregate marginal impressions equals the lost revenue on aggregate infra-marginal impressions. As in Section 1.3, the revenue gain from increased aggregate marginal impressions is higher when time use lost due to higher ad load on j is diverted to higher time use on platform $-j$, which also contributes to revenue. Unlike in Section 1.3, the merged platform calculates these aggregates by summing across heterogeneous individuals. Furthermore, ad load may be higher or lower than the social optimum because time use is unpriced, and lost revenue on aggregate infra-marginal impressions may not equal the marginal aggregate user welfare cost of ads.

In a separated equilibrium without duplication, platforms set ad load ignoring: (i) lost revenue on infra-marginal impressions on the other platform (a Cournot externality); and (ii) time use diversion to the other platform. As in Section 1.3, (i) increases ad load relative to the merged equilibrium, (ii) decreases it if platforms are substitutes, and the net effect is ambiguous. However, as in Section 1.4.2, these differences only apply for multi-homers with positive time use on both platforms. Therefore, the importance of each effect depends on user overlap. Ad load may be higher or lower than in the merged equilibrium, and therefore closer or further from the social optimum.

In a separated equilibrium with duplication, platforms again ignore time use diversion to the other platform. Because the market for ads is not integrated, there is no Cournot externality. Rather, duplication incentivizes higher ad load relative to the merged equilibrium, due to the inframarginal effect and business stealing effects described in Section 1.4.2. Therefore, ad load may be higher or lower in the separated equilibrium than the merged equilibrium.

However, because the separated equilibrium has inefficiently duplicated ad impressions, comparing ad load relative to the social planner benchmark is not sufficient to make welfare judgements. Even if ad load were identical to the social planner's choice, duplicated impressions in the separated equilibrium would lower advertiser surplus and hence overall welfare relative to the planner case. We work through these issues explicitly in Section 5.

The full model contains many positive and normative ambiguities about the effects of an antitrust policy that separates jointly-owned platforms. We now parameterize the model and take it to data to resolve those ambiguities.

2 Experimental Designs

We use a series of randomized experiments to generate empirical facts relevant to the model from Section 1 that we will later use to estimate the model parameters. We reanalyze two existing experiments to provide new evidence on three key user-side factors: the joint distribution of time-on-platform, the diversion ratio, and the price elasticity. We run a new experiment to estimate a central advertiser-side parameter relevant for counterfactuals: the change in click-through rates due to multiple impressions of the same advertisement. This section describes experimental designs, with results presented in Section 3.

2.1 User-side: Time use overlap, diversion ratio, and price elasticity

Facebook and Instagram Election Study ("FIES"). The first randomized experiment we consider is the 2020 Facebook and Instagram Election Study (Allcott et al. 2024), henceforth "FIES." We present results from Allcott et al. (2024) that have been disclosed through legal and privacy review and are relevant for the present paper. Allcott et al. (2024) use data from two randomized experiments, one that paid Facebook users to deactivate Facebook, and another that paid Instagram users to deactivate Instagram. We say that Facebook and Instagram, respectively, are the "focal platform" in each experiment.

From August 31 to September 12, 2020, Meta placed survey invitations at the top of the news feeds for a stratified random sample of 10,597,957 Facebook users and 2,633,497 Instagram users. Consenting participants were paid to complete a series of surveys and were also invited to a passive tracking sample where their mobile app use and desktop web browser use would be recorded. The final samples we use comprise 23,415 Facebook users and 21,249 Instagram users who were willing to deactivate the focal platform for a payment of \$25 per week, who completed the first two surveys, and whose survey responses could be linked to Meta's on-platform data. Approximately 25 percent of participants also consented to passive tracking.

A randomly selected Control group (comprising 73 percent of the sample) was paid \$25 if they did not log into their focal platform for the one week between September 23 and the end of the day on September 29. A randomly selected Deactivation group (comprising the remaining 27 percent of the sample) was paid \$150 if they did not log into their focal platform for the six weeks between

September 23, 2020, and the end of the day on November 3 (U.S. election day). The "treatment period" is the additional five weeks from September 30–November 3 when the Deactivation group was being paid to avoid logging in, while the Control group was not.

For each participant, Allcott et al. (2024) observe normalized daily time on Facebook and Instagram for each day of the baseline period through the end of 2020, as recorded by Meta. For each participant in the passive tracking sample, Allcott et al. (2024) observe daily time spent on each mobile app and daily count of impressions on each web domain. Allcott et al. (2024) also observe demographics, including income, education, age, gender, and race/ethnicity. Allcott et al. (2024) weight the final samples using weights constructed to make the samples representative of U.S. focal platform users (with baseline use over 15 minutes per day) on race, political party, education, and measures of baseline account activity, including number of friends (on Facebook) or accounts followed (on Instagram), number of days logged into the focal platform during the baseline period, and terciles of time spent on the focal platform during the baseline period.

Digital Addiction ("DA"). The second randomized experiment we study is Digital Addiction (Allcott, Gentzkow, and Song 2022), henceforth "DA". From March 22 to April 8, 2020, the authors recruited participants using Facebook and Instagram ads that were shown to 3,271,165 unique users. Consenting participants were paid to complete a series of surveys and install an app called Phone Dashboard that recorded their mobile app use. The final sample we use comprises 2,053 people were were randomized and informed about their treatment conditions on May 3, 2020.

Beginning May 3, a randomly selected Limit group was given access to functionality in Phone Dashboard that allowed them to set daily screen time limits for each app on their phone. A randomly selected Bonus group was paid to reduce their use of six apps (Facebook, Instagram, Twitter, Snapchat, web browsers, and YouTube, henceforth "FITSBY") over the 20 days beginning May 25. Specifically, users were told that they would be paid a "Screen Time Bonus" of \$50 for every hour they reduced their average FITSBY screen time over the 20-day period, relative to a benchmark set above their baseline use. For example, a user with a benchmark of 3 hours per day who had a daily average FITSBY screen time of 2.5 hours over the 20-day period would earn \$25. The Bonus treatment amounts to a price of \$2.50 per hour of screen time on the FITSBY apps during the 20-day period.

For each participant, we observe daily time spent on each mobile app from the beginning of the baseline period on April 12, 2020, through the end of the study on July 26, 2020. We do not apply sample weights to these data.

2.2 Advertiser-side: Loss from duplication.

We ran a new experiment on Meta to estimate how duplicated impressions impact click-through rates. This parameter is important for counterfactuals in our model, and plays a central role in many existing theoretical models of ad markets (e.g. Anderson et al., 2012; Athey, Calvano, and Gans, 2018; Gentzkow et al., 2024). Despite this, there are no (to our knowledge) direct estimates of this parameter in the literature. [3]We describe the experimental design here and present preliminary pilot results in Appendix B.1.

An ideal experiment would randomly vary the number of ads a user sees for a random sample of Meta advertising campaigns. We cannot implement this experiment outside of Meta, but Meta's functionalities allow us to approximate it as an outside advertiser. Meta's Ads Manager allows advertisers to retarget audiences initially impressed by another campaign, a feature we used to identify fixed user audiences. We then the varied whether a given audience saw one or two identical campaigns, and use this variation to estimate the change in clicks per ad impression.

With this approach, it is important to run a representative sample of ads, at representative ad intensity, so as to achieve typical ad performance. To do so, we designed ad campaigns for 15 advertisements spanning five product categories. We chose product categories to reflect the top categories by ad spending from the 2024 SensorTower Digital Market Index (SensorTower 2024).[4] We created separate Facebook pages that run ads recommending "product picks" within each category. We picked products to advertise based on the top product by the top-three advertisers within each category. We piloted one ad for the consumer packaged goods category.

Meta's functionality allows us to restrict targeting to a fixed user audience and estimate the number of unique users impressed by multiple campaigns. However, it does not guarantee that all users in an audience are actually impressed, nor does it give us precise control over ad frequency. This complicates estimation of average click-through rates, because for users more likely to click, Meta is more likely show ads and run ads at a higher frequency. It also makes it challenging to estimate the loss from duplication – unless all users are impressed by both campaigns, we cannot ensure that all impressions are duplicated. We overcome these challenges by experimentally varying ad budget, which traces out the relationship between the click-through rate and the fraction of the audience impressed, and whether the ad objective is to maximize clicks or maximize reach, which provides variation in frequency to trace out its relationship to the click-through rate. We use these relationships to control for different fractions of the audience impressed and different frequencies across treatment and control conditions.

For each ad, we ran an initial set of ads to identify five similarly-sized, non-overlapping audiences of US users aged 18-65. The following week, we randomly assigned these audiences to one of five treatment conditions. Across treatment conditions, we varied: (i) the daily budget to estimate the effect of reach on performance; (ii) the ad objective, either to maximize clicks or to maximize reach, to estimate the effect of frequency on performance; and (iii) the number of campaigns targeting the audience to estimate duplication loss. See Appendix B.1 for details on product selection, ad creatives, and implementation of the pilot.

[3]There are some structural estimates outside of digital competition settings. For example, Gentzkow, Shapiro, and Sinkinson (2014) estimate diminishing returns in historical newspaper markets based on the correlation between the political affiliation of incumbent newspapers and political affiliation of entrants, with the logic that high diminishing returns will discourage entry among ideologically-similar papers.

[4]The top five categories by spending are shopping, consumer packaged goods, media and entertainment, health and wellness, and financial services. Because Meta restricts financial services advertisements, we replace it with food and dining services, the sixth-highest category by spending.

3 Empirical Facts

3.1 Time Use and Overlap

As described in Section 1, an initial key statistic that determines the market effects of mergers or separations is the extent of overlap across users. Figure 2 is a heat map describing the joint distribution of Facebook and Instagram time use in the DA baseline data. The histograms at the top and right present the marginal distributions of Facebook and Instagram use, respectively. About 9 percent of Instagram users are single-homers (i.e., use zero Facebook), while the remaining 91 percent are multi-homers. About 30 percent of Facebook users are single-homers (i.e., use zero Instagram), while the remaining 70 percent are multi-homers.

3.2 Diversion Ratio

A second key statistic for merger analysis is the user-side diversion ratio between the two platforms. The FIES experiment allows Allcott et al. (2024) to estimate that directly, by estimating the effect of Facebook deactivation on Instagram use as well as the effect of Instagram deactivation on Facebook use. Below, we reproduce results previously reported in (Allcott et al., 2024).

Figure 3 presents background evidence, documenting Facebook and Instagram use in the respective Facebook and Instagram samples. The dark grey period (from September 23–29) indicates the week when both Deactivation and Control groups were being paid to deactivate. The light grey period (from September 30–November 3) indicates the five week treatment period when only the Deactivation group was being paid to deactivate. During the treatment period, about 15–20 percent of the Deactivation group would use the focal platform on any given day, compared to about 90 percent of the group.

We would like to estimate the effect of fully deactivating platform j on use of other app j'. However, Figure 3 shows that the experiment involved imperfect compliance: not all participants in the Deactivation group stayed fully deactivated. Allcott et al. (2024) thus analyze the experiment as a randomized encouragement design, using an instrumental variables estimator. Define $T_{ij,d}$ as person i's average daily use of platform j during the deactivation treatment period, and define $\hat{T}^C_{j,d}$ as the the Control group average. Allcott et al. (2024) define a deactivation compliance variable $\tilde{D}_i := \left(\hat{T}_{j,d} - T_{ij,d}\right)/\hat{T}_{j,d}$. $\tilde{D}_i = 1$ for participants who never use j during the treatment period, and $\tilde{D}_i = 0$ for participants with usage equal to the Control group average.

Define D_i as a deactivation group indicator, and define $T_{ij',0}$ and $T_{ij',d}$ as use of j' during the baseline and treatment periods, respectively. Allcott et al. (2024) estimate

$$T_{ij',d} = \tau^{Dj}_{j'} \tilde{D}_{ij} + \beta T_{ij',0} + \epsilon_{ij}, \tag{18}$$

instrumenting for $\tilde{D}_{ij}$ with the deactivation group estimator D_i. The first stage effect on $\tilde{D}_i$ measures the extent of compliance, and $\tau^{Dj}_{j'}$ is the local average treatment effect of fully deactivating j on use of j', for people induced to deactivate by the \$150 payment.

Panels (a) and (b) of Figure 4 present the local average treatment effects of Facebook and Instagram deactivation on use of other mobile apps, as measured in the passive tracking data. The Facebook and Instagram Control group participants in the passive tracking sample spent about 45 and 18 minutes per day, respectively, on the Facebook and Instagram mobile apps during the treatment period. Panel (a) shows that deactivating Facebook increases Instagram use by about 2 minutes per day, implying a diversion ratio of $\hat{\tau}_I^D \approx 2/45 \approx 0.044$. Panel (b) shows that deactivating Instagram has no statistically significant effect on Facebook use, implying a diversion ratio of $\hat{\tau}_F^D \approx 0$. The estimated effects on other social apps are relatively large (around 7–8 minutes per day).

3.3 Price Response

A third key statistic for evaluating effects on consumer surplus is the user price response. While we do not consider counterfactuals with positive prices, the price response is required to quantify the effects of ad load changes on consumer surplus. We identify the price response by estimating the effect of the DA experiment Screen Time Bonus on Facebook and Instagram time use. For these analyses, we use only the Limit Control group, thereby excluding people who had access to screen time limit functionality.

Figure 5 presents daily average Facebook and Instagram use for the Bonus and Control groups (within the Limit Control group). The grey period (from May 25–June 13) indicates the 20-day period when the Bonus group was being paid to reduce social media use. During that time, the Control group Facebook and Instagram averages are 54 and 14 minutes per day, and the Bonus group averages are noticeably lower.

Figure 6 estimates the average effect of the Bonus for each week of the experiment. The estimates show that Facebook use dropped by about 22 minutes per day, or 41 percent relative to the Control group. Instagram use dropped by about 7 minutes per day, or 50 percent relative to Control.

To formally estimate the effect of the Bonus on Facebook and Instagram use, define B_i as a Bonus group indicator, and define $T_{ij,0}$ and $T_{ij,b}$ as use of j during the baseline and bonus periods, respectively. We estimate

$$T_{ij,d} = \tau_j^B \tilde{D}_{ij} + \beta T_{ij,0} + \epsilon_{ij}. \tag{19}$$

τ_j^B is the effect of the Bonus on use of j.

These are strikingly large price responses—they imply that 41–50 percent of Facebook and Instagram use is worth less than \$2.50 per hour to users. This means that any reductions in time on platform or increases in ad load will translate to relatively small consumer surplus losses.

4 Structural Estimation

We now impose additional structure on the model and estimate the parameters using a minimum distance estimator that minimizes the differences between empirical moments and model predictions. The user-side empirical results from Section 3 help identify the user-side demand parameters. We infer advertiser-side parameters from the modeled ad load first-order condition from Section 1.5.

4.1 Setup

Define y_i as income and n_i as numeraire good consumption, both in units of \$/day. Define b as the price per minute of time spent on social media; $b = 0$ normally, but $b = b^B$ in the DA experiment Bonus condition. We assume that users maximize quadratic utility

$$U_i(\boldsymbol{T}_i) = \underbrace{\sum_j \left[(\xi_{ij} - \gamma_j \alpha_j) T_{ij} - \sigma_j T_{ij}^2/2\right] + \rho T_{i1} T_{i2}}_{\text{quadratic utility from time on platform}} + \underbrace{n_i}_{\text{numeraire}}, \tag{20}$$

subject to budget constraint $y_i = n_i + b \sum_j T_{ij}$.

We assume that α_j, σ_j, γ_j, and ρ are homogeneous across consumers, so all time use heterogeneity arises from differences in ξ_{ij}. Under this functional form, the average and marginal disutility for ads are the same, so there is no Spence (1975) quality distortion.

Maximizing utility gives users' choice of time use on platform j as a function of time on the other platform j':

$$T_{ij} = \frac{\xi_{ij} - \gamma_j \alpha_j - b + \rho T_{ij'}}{\sigma_j}. \tag{21}$$

Platform j single-homers exogenously have $T_{ij'} = 0$. Let $k \in \{s, m\}$ index single-homer and multi-homer user types, and let μ_j be the share of j's users that are multi-homers. Define ξ_{kj} as the mean of ξ_{ij} for platform j's user type k. Average time on platform j for single-homers, multi-homers, and all users are, respectively:

$$T_{sj} = \frac{\xi_{sj} - \gamma_j \alpha_j - b}{\sigma_j} \tag{22}$$

$$T_{mj} = \frac{(\xi_{mj} - \gamma_j \alpha_j - b) + \left(\xi_{mj'} - \gamma_j \alpha_{j'} - b\right) \cdot \rho / \sigma_{j'}}{\sigma_j - \rho^2 / \sigma_{j'}} \tag{23}$$

$$T_j = (1 - \mu_j) T_{sj} + \mu_j T_{mj} \tag{24}$$

We consider three treatment conditions, indexed by g:

- B: the DA experiment Bonus group, modeled by setting $b = b^B$ in equations (22)–(24).

- Dj: the FIES experiment Facebook and Instagram Deactivation groups, modeled by setting $T_{ij} = 0$ and $b = 0$. This gives $T_{mj'}^{Dj} = \frac{\xi_{mj'} - \gamma_j \alpha_{j'}}{\sigma_{j'}}$.

- C: Control, modeled by setting $b = 0$ in equations (22)–(24).

Define T_j^g as the modeled average time use in treatment condition g, and define T_{kj}^g as that average for user type k. Define $\hat{T}_j^g$ and $\hat{T}_{kj}^g$ as the observed empirical analogues.

4.2 Distance Functions

Define $\mathbf{\Theta} \coloneqq \left\{ \{\xi_{kj}\}, \{\sigma_j\}, \rho, \{\gamma_j\}_j, \eta, \eta_0, \alpha_I \right\}$ as the vector of 12 structural parameters to be estimated. Our estimator minimizes the sum of squared errors between modeled statistics (such as diversion ratios and price responses) and their empirical analogues. We now describe the distance functions expressing these errors.

Tables 1 and 2 present the exogenous parameters and empirical moments. We set $m = 1$ as a normalization. Note that the number of advertisers A is a normalization that impacts units of estimated η, η_0 but does not impact incentives faced by platforms in either combined or separated equilibria.

Control group use. There are four control group use moments: average Facebook and Instagram use for single-homers and multi-homers. The distance functions are the difference between modeled and empirically observed use:

$$h_{kj}^C(\mathbf{\Theta}) = T_{kj}^C - \hat{T}_{kj}^C. \tag{25}$$

We construct the empirical moments from the DA experiment. As shown in Table 2, for single-homers and multi-homers, respectively, average Facebook use is 0.82 and 0.75 hours per day, and average Instagram use is 0.33 and 0.30 hours per day.

These average use moments will be most informative about the value of the average demand shifters ξ_{kj}.

Price response. There are two price response moments: percent reductions in Facebook and Instagram use from the DA experiment Bonus condition. The distance functions are the difference between modeled and empirically observed percent reductions:

$$h_j^B(\mathbf{\Theta}) = \frac{T_j^B - T_j^C}{T_j^C} - \frac{\hat{\tau}_j^B}{\hat{T}_j^C}. \tag{26}$$

We normalize the bonus effects by control group usage because participants in the DA experiment were relatively heavy users, which could make the non-normalized effects larger than they would be for average users.

We construct the empirical moments from the DA experiment; $\hat{\tau}_j^B$ is estimated in equation (19). As shown in Table 2, the bonus reduced Facebook and Instagram use by 38 and 33 percent, respectively.

Note that the DA experiment bonus also changed the appeal of the outside option, because it subsidized reductions from the six FITSBY apps, not just Facebook and Instagram. However, since Facebook and Instagram comprise 80 percent of FITSBY consumption in the DA baseline data, the modeled moment in equation (26) assumes no change in the appeal of the outside option. Since if anything, the other four apps appear to be substitutes for Facebook and Instagram, subsidizing reductions in those other four apps increases Facebook and Instagram use, biasing the estimated price response toward zero. Since we find that the price response is already quite large, any bias strengthens our results that any effects on consumer surplus are relatively small.

The price responses will be most informative about the diminishing marginal utility parameters σ_j. To see this, notice from equation (21) that single-homers' modeled response to the bonus is $T^B_{sj} - T^C_{sj} = -b^B/\sigma_j$. Multi-homers' modeled price response is that same quantity adjusted for the change in $T_{ij'}$.

Diversion ratio. There is one diversion ratio moment: the average diversion ratio from Facebook and Instagram deactivation. The distance function is

$$h^D(\mathbf{\Theta}) = \frac{1}{2}\sum_j \left[\frac{T^{Dj}_{j'} - T^C_{j'}}{T^C_j} - \frac{\hat{\tau}^{Dj}_{j'}}{\hat{T}^C_j}\right]. \tag{27}$$

We construct the empirical moment from the FIES experiments; $\hat{\tau}^{Dj}_{j'}$ is estimated in equation (18), and $\hat{T}^C_j$ is the Control group average use during the treatment period. As shown in Table 2, the diversion ratios are very close to zero.

The deactivation treatment effect $\hat{\tau}^D_{j'}$ is local to the people induced to deactivate by the \$150 payment. In theory, the average treatment could be different for the full population that includes the "never-takers" not induced to deactivate by the payment. One might hypothesize that the never-takers are unwilling to deactivate j partially because they don't feel they have good online substitutes, and thus that the diversion ratio to social media platform j' would be smaller. Any such bias would strengthen our finding that the diversion ratios are quite small.

The diversion ratios will be most informative about the substitution parameter ρ. To see this, notice from equation (21) that for multi-homers, the effect of deactivating platform j (i.e., exogenously setting $T_{ij} = 0$) on platform j' time use is $T^{Dj}_{mj'} - T^C_{mj'} = -\frac{\rho T^C_{mj}}{\sigma_{j'}}$. Since platform j single-homers always have $T_{ij'} = 0$ and multi-homers comprise share μ_j of platform j's users, the effect of deactivating j on platform j' time use among platform j users is $T^{Dj}_{j'} - T^C_{j'} = -\mu_j \frac{\rho T^C_{mj}}{\sigma_{j'}}$.

Ad elasticity. There are two ad elasticity moment, corresponding to modeled ad load elasticities on Facebook and Instagram. The distance function is

$$h^A_j(\mathbf{\Theta}) = \frac{\partial T_j}{\partial \alpha_j}\frac{\alpha_j}{T_j} - \widehat{\frac{\partial T}{\partial \alpha}\frac{\alpha}{T}}. \tag{28}$$

We import the empirical moment from Brynjolfsson et al. (2024), which evaluated a 9-year

Facebook experiment where a random 0.5% subset of users never experienced ads.[5] On the basis of their Figure 2, we assume that the long-run elasticity of time-on-platform to ad load is $\widehat{\frac{\partial T}{\partial \alpha}\frac{\alpha}{T}} = -0.094$, with a standard error of 0.019[6]. We assume that the ad elasticity is the same on both platforms.

The ad elasticities will be most informative about the ad disutility parameter γ_j. To see this, notice from equation (21) that single-homers' modeled ad response is $\frac{\partial T_{sj}}{\partial \alpha_j} = \frac{-\gamma_j}{\sigma_j}$. Multi-homers' modeled price response is that same quantity adjusted for the change in $T_{ij'}$.[7]

Platform first-order condition. We do not currently have an estimate of Meta's price elasticity of advertising demand. Instead, we identify that parameter by assuming that Meta sets ad load to maximize revenues in our model—just as it is common to infer firms' marginal costs by assuming static Nash-Bertrand pricing, following Berry, Levinsohn, and Pakes (1995). Beginning with the merged equilibrium revenue function from equation (5) and substituting our parameterization of $H(\cdot)$, the platform's first-order conditions give

$$h_j^F(\mathbf{\Theta}) = 0 = \sum_i \left[\frac{\partial}{\partial \alpha_j}\left(\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})\right)\right]\cdot p_i\left(\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})\right)\cdot\left(1+\underbrace{\frac{\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})}{p_i\left(\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})\right)}\cdot\frac{\partial p_i}{\partial\left(\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})\right)}}_{\text{inverse demand price elasticity}}\right). \quad (29)$$

The expression for $\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})$ and $\frac{\partial}{\partial \alpha_j}\left(\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})\right)$ are known conditional on the user-side parameters, and the remaining terms depend on advertiser-side parameters. Conditional on $p_i(\cdot)$, these conditions identify the price elasticity of inverse advertiser demand. In addition, $T_{ij}(\boldsymbol{\alpha}) = T_{kj}(\boldsymbol{\alpha}) + e_{ij}$. Inspecting equation (29) makes clear that the first-order condition will depend on $\mathcal{E}_{kj}^2 \equiv E\left[e_{ij}^2 | i\in\mathcal{U}_k\right]$ for $k = s, m$ and $\mathcal{E}_{mFI} \equiv E\left[e_{i1}e_{i2}|i\in\mathcal{U}_m\right]$. We use the variance and covariance of time use from the DA experiment, equivalent to using nonparameteric estimates of the distribution of ξ_{ij}.

Average ad price. To pin down η, we also incorporate information from Facebook's average cost per impression observed on RevealBot during our study period, $\hat{P}_F \equiv \frac{1}{T}\sum_t \hat{p}_{Ft}$, and match this to our model counterpart, given by:

$$\frac{1}{I_F}\sum_{i\in\mathcal{U}_j}\alpha_F\cdot T_{iF}(\boldsymbol{\alpha})\cdot p_i = \frac{1}{I_F}\sum_{i\in\mathcal{U}_F}\alpha_F\cdot T_{iF}(\boldsymbol{\alpha})\cdot\eta\cdot\left(1+\eta_0-\frac{\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})}{A}\right) \quad (30)$$

where $I_F := \sum_{i\in\mathcal{U}_F}\alpha_F\cdot T_{iF}(\boldsymbol{\alpha})$. The distance function is therefore:

$$h^P(\mathbf{\Theta}) = \frac{1}{I_F}\sum_{i\in\mathcal{U}_F}\alpha_F\cdot T_{iF}(\boldsymbol{\alpha})\cdot p_i - \hat{P}_F \quad (31)$$

[5]The experiment estimates the elasticity of average time use with respect to ad load, which is what we model above.

[6]We back out standard errors by assuming a standard normal distribution and we assume no covariance of the ads and no-ads groups to be able to combine the standard errors of the groups

[7]The overall modeled ad elasticity is $\frac{\partial T_j}{\partial \alpha}\frac{\alpha}{T_j} = \sum_j -\gamma\cdot\left[\frac{1-\mu}{\sigma_j}+\frac{\mu}{\sigma_j-\rho^2/\sigma_{j'}}\right]\cdot\frac{\alpha}{T_j^C}$.

Our user-side estimates come from the 2020 Facebook deactivation experiment, which covers 2020Q4. We therefore calculate average price per click over this period by scraping weekly data reported on RevealBot's website. RevealBot reports prices separately for Facebook and Instagram. Equation (31) also depends on variance and covariance of time use.

Ad load. We assume that we know ad load on Facebook with certainty, but do not observe ad load on Instagram. Therefore, α_F is known, but α_I is a parameter to be estimated. The first-order conditions, combined with the average ad price moment, constitute 3 equations with 3 unknowns, conditional on user-side parameters (η, η_0, α_I). Intuitively, Meta's first-order condition for Facebook, combined with observed ad load, average price, and user-side parameters, identifies the price elasticity of advertising demand. The first-order condition for Instagram then uses the user-side parameters and price elasticity of demand to back out α_I.

Loss from duplication. Advertiser's losses from inefficient duplication of impressions depends on ζ_j, which controls the decrease in click-through rates for duplicated campaigns. For person i and advertiser a, the click-through rate for impressions in the first campaign is ω_{ia}, and the click-through rate for impressions in the second campaign is $(1 - \zeta_i) \cdot \omega_{ia}$. By definition, the average click-through rate for the first campaign is $E[\omega_{ia}]$. If all users are shown an identical campaign twice, the average click-through rate across both campaigns is $\frac{1}{2}E[\omega_{ia} + (1-\zeta_i)\omega_{ia}]$. The percentage decrease in click-through rates for a duplicated campaign is therefore:

$$\frac{\frac{1}{2}E\left[(2-\zeta_i)\omega_{ia}\right] - E[\omega_{ia}]}{E[\omega_{ia}]} = \frac{1}{2}\frac{E\left[-\zeta_i\omega_{ia}|S\right]}{E[\omega_{ia}|S]} \tag{32}$$

Since the regularity condition described in Section 1.1 must hold pointwise for each i, and ζ_j depends the full distribution of ζ_i among multi-homers since it takes a time-use weighted average, we must parameterize ζ_i. We assume that $(1 - \zeta_i) = \kappa \cdot p_i(\boldsymbol{\alpha}^m)/(\eta \cdot (1 + \eta_0)$ for $\kappa \in [0, 1]$. We estimate κ with the distance function:

$$h^L(\boldsymbol{\Theta}) = \left(\left[\frac{\kappa}{2} \cdot \frac{1}{N}\sum_i \frac{p_i(\boldsymbol{\alpha})}{\eta \cdot (1+\eta_0)}\right] - \frac{1}{2}\right) - \hat{L} \tag{33}$$

where $\hat{L}$ is the experimental estimate from Section B.1 of the percentage decrease in average click-through rates due to duplication.

4.3 Estimation

Define $\boldsymbol{h}(\boldsymbol{\Theta}) := \left\{\left\{h^C_{kj}\right\}, \left\{h^B_j\right\}, h^D, \left\{h^A_j\right\}, \left\{h^F_j\right\}, h^P\right\}$ as the vector of distance functions described above. $\boldsymbol{\Theta}$ contains 12 structural parameters, and $\boldsymbol{h}(\boldsymbol{\Theta})$ contains 12 distance functions. Our estimate of $\boldsymbol{\Theta}$ minimizes the sum of squared distance functions:

$$\hat{\boldsymbol{\Theta}} = \arg\min_{\boldsymbol{\Theta}} \boldsymbol{h}(\boldsymbol{\Theta})' \boldsymbol{h}(\boldsymbol{\Theta}). \tag{34}$$

We construct standard errors using the Delta method. See Appendix B.2 for details.

For counterfactuals, we also need estimes of the the distribution of ξ_{ij}. We estimate this distribution non-parameterically. Let $\xi_{ij} = \xi_{kj} + \varepsilon_{ij}$ for $k \in \{s, m\}$. For each i in the DA control group, we invert Equation (21) given estimated parameters to estimate ε_{ij} for all i, j.[8] The resulting ε_{ij} are draws from distributions Ξ_m and Ξ_{sj}, where Ξ_m is the joint distribution of $(\varepsilon_{i1}, \varepsilon_{i2})$ for users in group $k = m$ and $\Xi_{sj}$is the distribution of ε_{ij} for users in group $k = s$ on platform j. We use observed draws of ε_{ij} to estimate $\hat{\Xi}_m$ and $\hat{\Xi}_{sj}$ using a Gaussian kernel. Define $\hat{\boldsymbol{\Xi}} := \left(\hat{\Xi}_m, \hat{\Xi}_{s1}, \hat{\Xi}_{s2}\right)$ as a tuple collecting the estimated distributions.

Table 3 presents preliminary estimates. Instagram time use has much steeper curvature than Facebook time use. This means Instagram time use is "stickier," and will respond less to changes in ad load than time use on Facebook. For this reason, ad load estimated from the first-order condition is higher on Instagram than Facebook, although the estimate is very imprecise. The diversion parameter ρ is negative but small, indicating that the platforms are weak substitutes. We calculate the aggregate elasticity of advertiser demand as -1.2 on Facebook and -1.22 on Instagram. [9] Finally, our pilot experiment estimating loss from duplication indicates that $\kappa = 0.68$. This implies that the second impression is worth at most 68 percent of the value of the first impression, and implies that in the merged equilibrium, $(\zeta_F, \zeta_I) = (0.63, 0.64)$.

5 Counterfactual Simulations of Facebook-Instagram Separation

In this section, we simulate a Facebook-Instagram separation. In the separated equlibrium, platforms independently set ad load. We assume they retain their respective ad targeting systems, meaning that perceived ω_{ia} is the same for i on both platforms. Since each platform cannot observe whether user i is impressed by their rival, there may be wasteful duplication.

Counterfactual ad load. Under Facebook-Instagram separation, reaction functions are (substituting in advertiser-side parameters):

$$\alpha_j^{*,s}\left(\alpha_{-j}; \boldsymbol{\Theta}, \boldsymbol{\Xi}\right) := \arg\max_{\alpha_j} \left(1 - \zeta_j(\boldsymbol{\alpha}) \cdot O'_{aj}\left(\boldsymbol{\alpha}\right)\right) \cdot \eta \left(\sum_i \alpha_j T_{ij}(\boldsymbol{\alpha}) \cdot \left(1 + \eta_0 - \frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{A}\right)\right) \quad j = 1, 2 \tag{35}$$

The counterfactual equilibrium is (α_1^s, α_2^s) the fixed point where $\alpha_j^{*,s}(\alpha_{-j}^s; \cdot) = \alpha_j^s$ for $j = 1, 2$. Note that evaluating Equation (35) requires calculating marginal overlap $O'_{aj}(\boldsymbol{\alpha})$, which involves integrating over a transformation of the distribution Ξ_m where the transformation depends on α_j and α_{-j}.

[8]The minimum distance estimator implicitly relies on the distribution of ε_{ij} because the platform first-order condition depends on variance and coviarance of e_{ij}. However, since we estimate ε_{ij} pointwise, without imposing any functional form, all that matters for platform incentives are the variance and covariance of e_{ij}, which we can estimate directly.

[9]We define the aggregage elasticity of advertiser demand by analogy to the elasticity of advertiser demand in a one-sided market with homogeneous users. See Appendix C.2.3 for details.

Welfare in counterfactual equilibrium. Let $\boldsymbol{\alpha}^m$ and $\boldsymbol{\alpha}^s$ represent ad load in merged and counterfactual separated equilibrium, respectively, with time use $\boldsymbol{T}_i^m$ and $\boldsymbol{T}_i^s$. The change in consumer surplus is:

$$\Delta CS := \sum_i U_i(\boldsymbol{T}_i^s) - U_i(\boldsymbol{T}_i^m) \tag{36}$$

The change in platform surplus is:

$$\Delta PS := \sum_j R_j^s(\boldsymbol{\alpha}^s) - R^m(\boldsymbol{\alpha}^m) \tag{37}$$

Advertiser surplus in the merged equilibrium intergrates under the advertiser demand curve for each user and sums across users:

$$AS^m(\boldsymbol{\alpha}^m) = \sum_i Am \cdot \int_{p_i(\boldsymbol{\alpha}^m)}^{(\pi\omega)^{\max}} x dH(x) \tag{38}$$

where $p_i(\boldsymbol{\alpha})$ is given by Equation (4). Advertiser surplus in the counterfactual equilibrium changes due to both endogenous changes in price, as well as inefficient duplication of ads. In Appendix C.1, we show that in the counterfactual equilibrium:

$$AS_j^s(\boldsymbol{\alpha}^s) = \overbrace{\sum_{i \in \mathcal{U}_j} Am \cdot \int_{p_{ij}(\boldsymbol{\alpha}^s)}^{(\pi\omega)^{\max}} x dH(x)}^{\text{surplus — no duplication}} - \overbrace{\zeta_j \cdot \sum_i \int_{\underline{p}_i(\boldsymbol{\alpha}^s)}^{(\pi\omega)^{\max}} x dH(x)}^{\text{duplication loss}} \tag{39}$$

$$\text{where } \underline{p}_i(\boldsymbol{\alpha}^s) := \max_k \; H_i^{-1}\left(1 - \frac{\alpha_k T_{ik}(\boldsymbol{\alpha}^s)}{Am}\right)$$

The change in advertiser surplus is therefore $\Delta AS := \sum_j AS_j^s(\boldsymbol{\alpha}^s) - AS^m(\boldsymbol{\alpha}^m)$. Appendix C.1 provides explicit formulas given our parameterizations.

Main counterfactual results. Table 4 presents preliminary results. Panel A shows effects on market outcomes. Ad load increases on both platforms by a similar amount in levels. Time use on both platforms declines, although not by much. Because the Instagram curvature parameter σ_j is much higher, time use falls by less on Instagram in response to greater ad load than time use on Facebook does. Ad prices fall significantly, both due to duplication (advertisers demand a lower price to fill the same number of ad slots) and greater ad supply.

Panel B shows how these market effects impact surplus. Consumer surplus falls slightly. Advertiser surplus rises by about 32%. However, surplus would have risen by 61% with no duplication. The wedge is due to $\zeta_j > 0$: at counterfactual equilibrium ad load, $(\zeta_F, \zeta_I) = (0.64, 0.64)$, implying that the revenue loss from expected overlap is about 60%. This demonstrates that duplication produces significant inefficiencies that are quantitatively relevant for welfare. Platform surplus falls by 4.5% on Facebook at 24.3% on Instagram.

The total surplus effects are quite modest, because losses from lower consumer and platform surplus offset gains from higher advertiser surplus. Total surplus falls by 0.9%, while advertiser and consumer surplus combined increases by 1.4%. Overall, the platform separation mostly transfers surplus from platforms to advertisers.

Counterfactual sensitivity. We now investigate the sensitivity of counterfactual results to our parameter estimates. We perturb one structural parameter at a time, leaving the rest unchanged; find the merged and separated equilibrium ad load under the new parameters; and calculate the change in total surplus from the merged to the separated equilibrium as a fraction of total surplus in the baseline merged equilibrium with original parameters. These exercises illustrate how our user and advertiser-side empirical estimates help resolve theoretical ambiguities about the welfare effects of a Facebook-Instagram separation.

Figure 7 shows the total surplus impact of a Facebook-Instagram separation under alternative user-side parameter estimates. The top panel demonstrates the importance of credibly estimating user diversion: were the platforms stronger substitutes (ρ more negative), then platform separation could increase total surplus. The top panel of Appendix Figure A1 explains why: if the platforms were stronger substitutes, separation would cause Instagram to cut ad load to compete for users, increasing user surplus by enough to overwhelm a lower loss in advertiser surplus (because Facebook does not cut ad load) and inefficient duplication of impressions.

The bottom panel of Figure 7 demonstrates the non-monotonic relationship between user aversion to ads and the total surplus impact of platform separation. When average γ_j goes to zero, meaning users do not care about ads, platform separation increases competitiveness of ad markets and increases total surplus. As users become more averse to ads, separation still increases ad load as platforms compete primarily on the advertiser side of the market, but the infra-marginal welfare loss on the user side overwhelms the advertisers' surplus gains. But once user ad aversion becomes sufficiently large, platforms begin to compete on the user side, shown in the bottom panel of Appendix Figure A1. User-side competition reduces separated equilibrium ad load, and the increased user surplus overwhelms the lost advertiser surplus.

Turing to advertiser-side parameters, Figure 8 presents the impact of user overlap and loss from duplication on the total surplus effects.[10] In the top panel, we vary the share of multi-homers as a share of total users, and allocate single homers to keep the ratio single-homers on Facebook and Instagram constant. Overlap has a minimal impact on total surplus: as overlap decreases, platforms compete less intensively on the advertiser side, meaning ad load increases by less, but losses from inefficient duplication and user disutility of ads also fall to partially compensate.

The bottom panel shows that the sign of the total surplus change from platform separation hinges on an appropriate estimate for the loss from duplicated impressions. The figure plots the total surplus change due to separation for alternative assumptions about the loss from duplication, where average ζ_j spans its entire admissable range (from $\kappa = 1$, implying low loss from duplication and low ζ_j, to $\kappa = 0$, implying high loss from duplication and high ζ_j). If loss from duplication were

[10] Appendix Figure A2 shows the effects of these alternative estimates on equilibrium ad load.

lower than our experimental estimates, platform separation could increase total surplus, because the increased advertiser surplus from higher ad load would overwhelm the user surplus loss. If we had assumed complete loss from duplication, plotted to the far right of the figure, we would have estimated much higher total surplus losses of 3%.

6 Conclusion

This paper introduces a new model of competition between two-sided media platforms with heterogeneous advertisers running targeted ads to impress heterogeneous users. The model highlights the distortions that occur when platforms are jointly owned, and characterizes important empirical parameters that govern whether anti-trust policy that separates ownership can mitigate those distortions.

Our theory emphasizes how jointly owned platforms coordinate ads across platforms to avoid inefficient duplication of impressions, but inefficiently restrict ad load to raise prices. Separating platform ownership increases competition, but also leads to inefficient duplication of impressions, since platforms can no longer coordinate. Competitive incentives in the separated ownership equilibrium depends on user overlap across platforms, user time use heterogeity, user diversion across platforms, and the advertiser price elasticity of demand.

We apply our framework to study the effects of separating two of Meta's leading platforms: Facebook and Instagram. First, we provide descriptive empirical evidence on user-side parameters from randomized experiments on Meta users. These experiments show that Facebook and Instagram are weak substitutes, implying limited incentives for platforms to restrict ad load to compete on the user side. Next, we use experimental moments to structurally estimate a parameterized version of our model. This allows us to infer advertiser demand elasticities based on Meta's current actions and a first-order condition for profit maximization.

We use the estimated model to simulate the effects of a Facebook-Instagram separation contemplated by several global anti-trust authorities. Preliminary estimates indicate that separation would mostly transfer surplus from platforms to advertisers, with a small loss for users. Total surplus would remain mostly unchanged, since large potential gains to advertisers through greater ad load are offset by inefficiencies due to wasted impressions, lost consumer surplus from more ads, and lost platform surplus. Our findings indicate that separation causes meaningful inefficiencies and has important redistributive, but perhaps not total welfare, effects.

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Table 1: **Exogenous Parameters**

Parameter	Formula	Value	Data source
FB ad load (ads/hour)	α_F	133	Authors' calculations
Share of FB users that are multi-homers	μ_F	0.68	DA
Share of IG users that are multi-homers	μ_I	0.88	DA
FB ad price ($/1000 impressions)	p_F	16	Revealbot
IG ad price ($/1000 impressions)	p_I	11	Revealbot
Number of US advertisers	A	40,000	MediaRadar
Impressions per campaign	m	1	Normalization

Notes: This table presents the exogenous parameters used to construct the distance functions described in Section 4. "DA" refers to the paper "Digital Addiction" (Allcott, Gentzkow, and Song, 2022).

Table 2: **Empirical Moments**

Parameter	Formula	Value	SE	Data source
Single-homer average FB use (hours/day)	$\hat{T}^C_{sF}$	1.15	0.09	DA
Multi-homer average FB use (hours/day)	$\hat{T}^C_{mF}$	1.07	0.05	DA
Single-homer average IG use (hours/day)	$\hat{T}^C_{sI}$	0.39	0.07	DA
Multi-homer average IG use (hours/day)	$\hat{T}^C_{mI}$	0.36	0.03	DA
FB Bonus response	$\frac{\hat{\tau}^B_F}{\hat{T}^C_F}$	0.38	0.04	DA
IG Bonus response	$\frac{\hat{\tau}^B_I}{\hat{T}^C_I}$	0.33	0.03	DA
FB-IG diversion ratio	$\frac{\hat{\tau}^{DF}_{mI}}{\hat{T}^C_{mF}}$	0.044	0.022	FIES
IG-FB diversion ratio	$\frac{\hat{\tau}^{DF}_{mF}}{\hat{T}^C_{mI}}$	0.00	0.11	FIES
Ad load elasticity	$\frac{\partial T_j}{\partial \alpha_j}\frac{\alpha_j}{T_j}$	-0.094	0.019	Brynjolfsson et al. (2024)
% CTR decrease from duplication	$\hat{L}$	0.27		Duplication loss pilot

Notes: This table presents the empirical moments used to construct the distance functions described in Section 4. "DA" refers to the paper "Digital Addiction" (Allcott, Gentzkow, and Song, 2022). "FIES" refers to the 2020 Facebook and Instagram Election Study (Allcott et al., 2024).

Table 3: **Parameter Estimates**

Parameter	Description	Units	Estimate	SE
ξ_{sF}	**FB single-homer demand intercept**	**\$/hour**	**7.9**	**0.88**
ξ_{mF}	**FB multi-homer demand intercept**	**\$/hour**	**7.5**	**0.81**
ξ_{sI}	**IG single-homer demand intercept**	**\$/hour**	**9.3**	**1.26**
ξ_{mI}	**IG multi-homer demand intercept**	**\$/hour**	**8.8**	**0.84**
σ_F	**FB curvature**	**\$/hour^2**	**8.4**	**0.90**
σ_I	**IG curvature**	**\$/hour^2**	**24.1**	**3.14**
ρ	**Cross-demand response**	**\$/hour^2**	**0.343**	**0.85**
γ_F	**FB ad load disutility**	**\$/ad**	**0.0080**	**0.0020**
γ_I	**IG ad load disutility**	**\$/ad**	**0.0058**	**0.01**
α_I	**IG ad load**	**ads/hour**	**235.3**	**404**
η	**Advertiser demand slope**	$\frac{\text{share of } A}{\text{\$/impression}}$	**1.20**	**0.69**
η_0	**Advertiser demand intercept**	**share of A**	**0.983**	**0.01**
κ	**Value of duplicated impression**	**1/\$**	**0.68**	

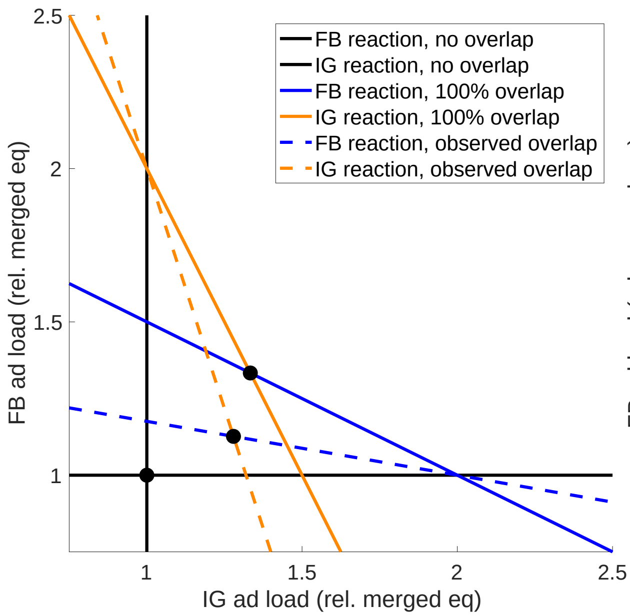
Notes: This table presents the parameter estimates from the estimation procedure described in Section 4.

Table 4: **Counterfactual Simulation Results**

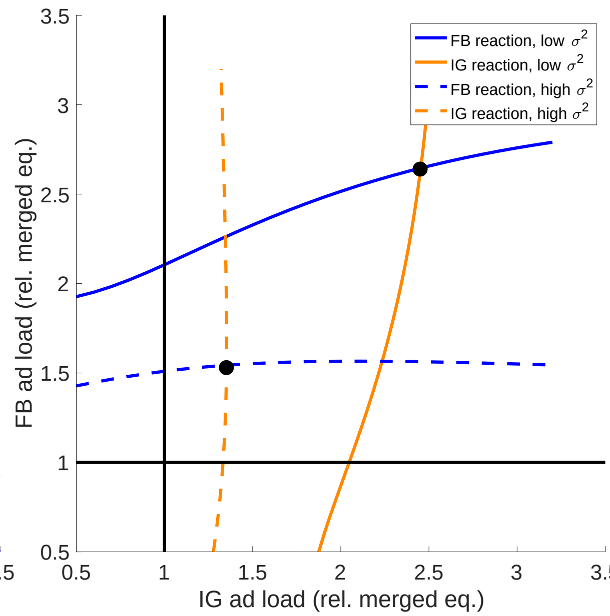
	(1) Baseline	(2) FB-IG separation: Δ from baseline	(3) FB-IG separation: %Δ from baseline
Panel A: Market Outcomes			
FB ad load (ads/hour)	133	12.0	9.0%
IG ad load (ads/hour)	235	9.0	3.8%
Average time on FB (hours)	0.77	-0.01	-1.5%
Average time on IG (hours)	0.30	-0.00	-0.7%
Average FB ad price (\$/1000 impressions)	12	-2	-15%
Average IG ad price (\$/1000 impressions)	11	-3	-28%
B. Surplus			
Consumer surplus (\$/user-year)	3,244	-28.6	-0.9%
Advertiser surplus (\$/user-year)	242	77.0	31.8%
if no duplication loss	242	155.9	64.4%
duplication loss (\$/user-year)	-	78.9	-32.6%
Advertiser + consumer surplus (\$/user-year)	3,486	48.3	1.4%
Platform surplus: FB (\$/user-year)	422	-28.6	-6.8%
Platform surplus: IG (\$/user-year)	296	-81.8	-27.7%
Total surplus (\$/user-year)	4,082	-36.0	-0.9%

Notes: This table presents the counterfactual simulation results. Baseline refers to the initial equilibrium, where Facebook and Instagram are merged. Columns 2 and 3 present the simulated effects of separating Facebook and Instagram.

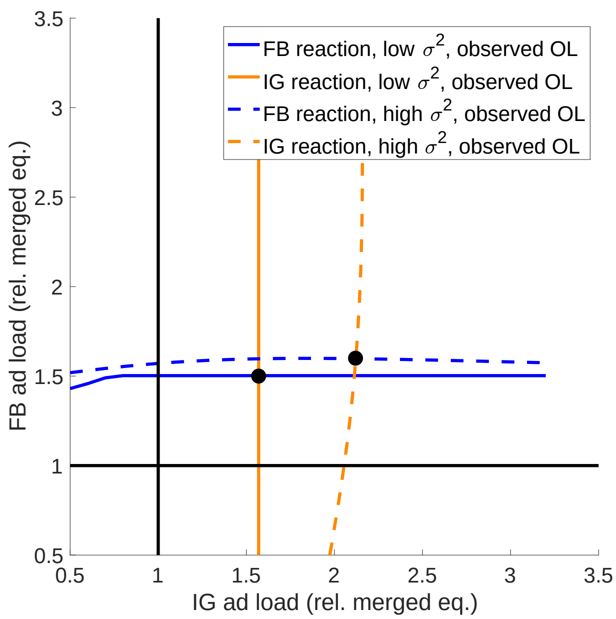
Figure 1: **Key Advertiser-Side Forces: Numerical Examples**



(a) **Partial Overlap, No Duplication**



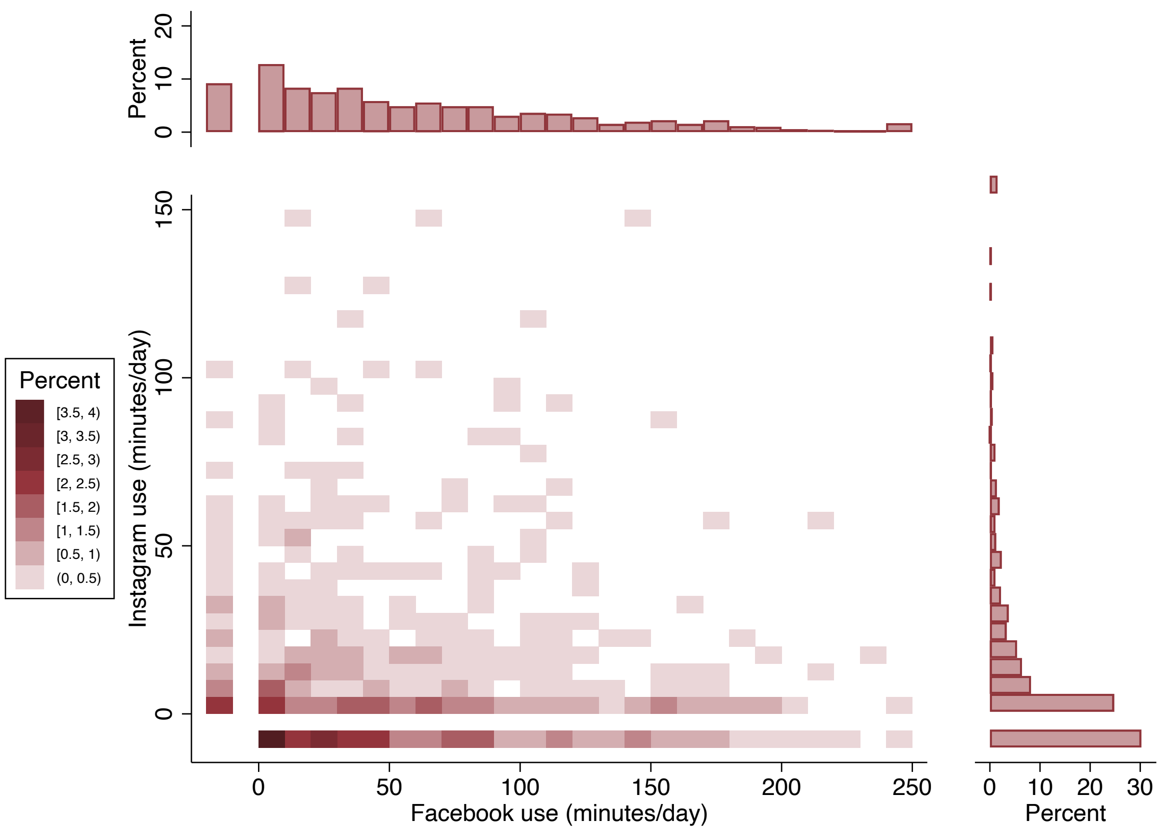
(b) **Full Overlap, With Duplication**



(c) **Partial Overlap and Duplication Together**

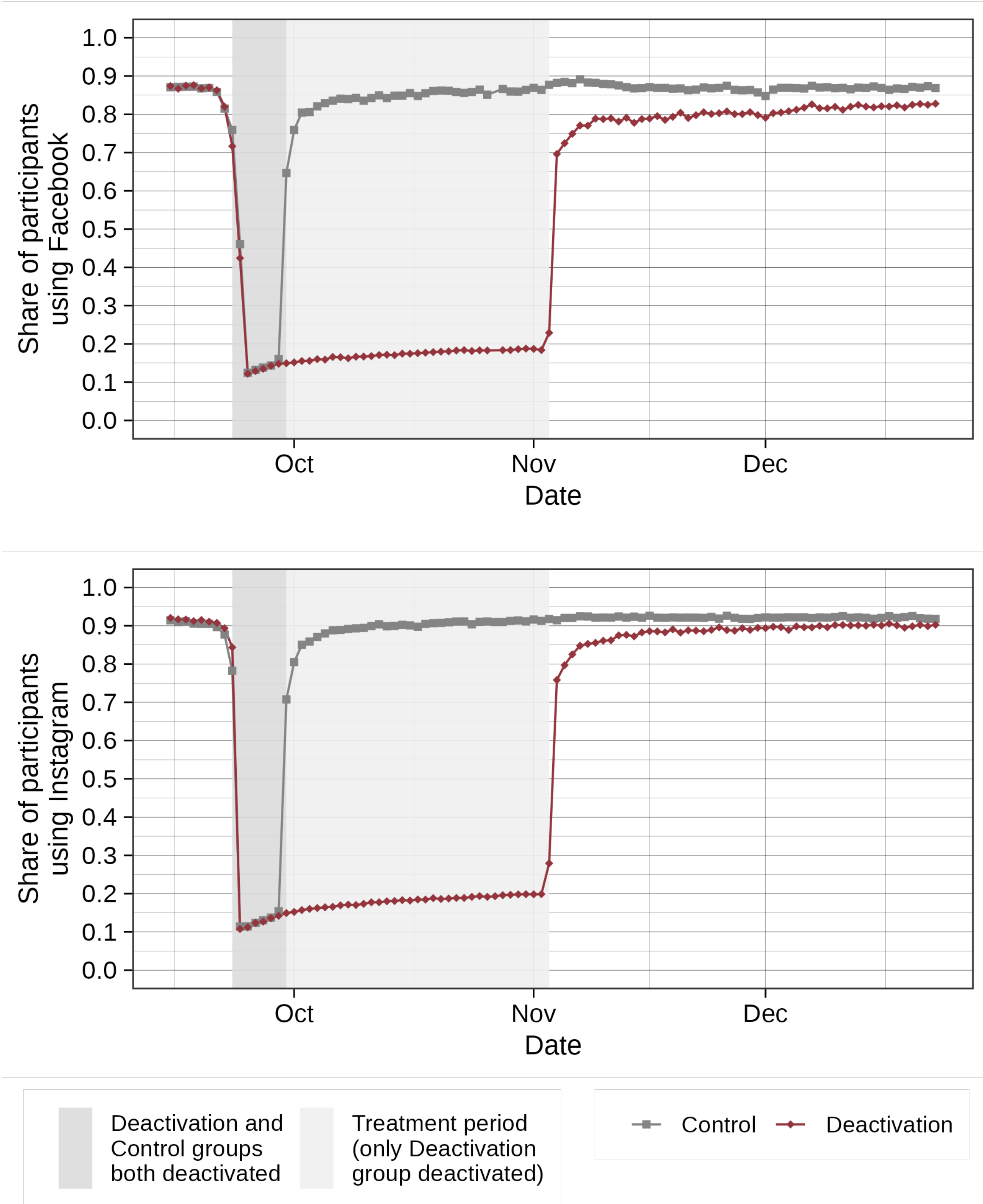
Notes: This figure presents numerical examples described in Section 1.4.3. Panel (a) plots reaction functions in an example with partial overlap but no duplication in separated equilibrium. Panel (b) plots reaction functions in an example with full overlap and duplication. Panel (c) plots reaction functions in an example with partial overlap and duplication.

Figure 2: **Time Use and Overlap**



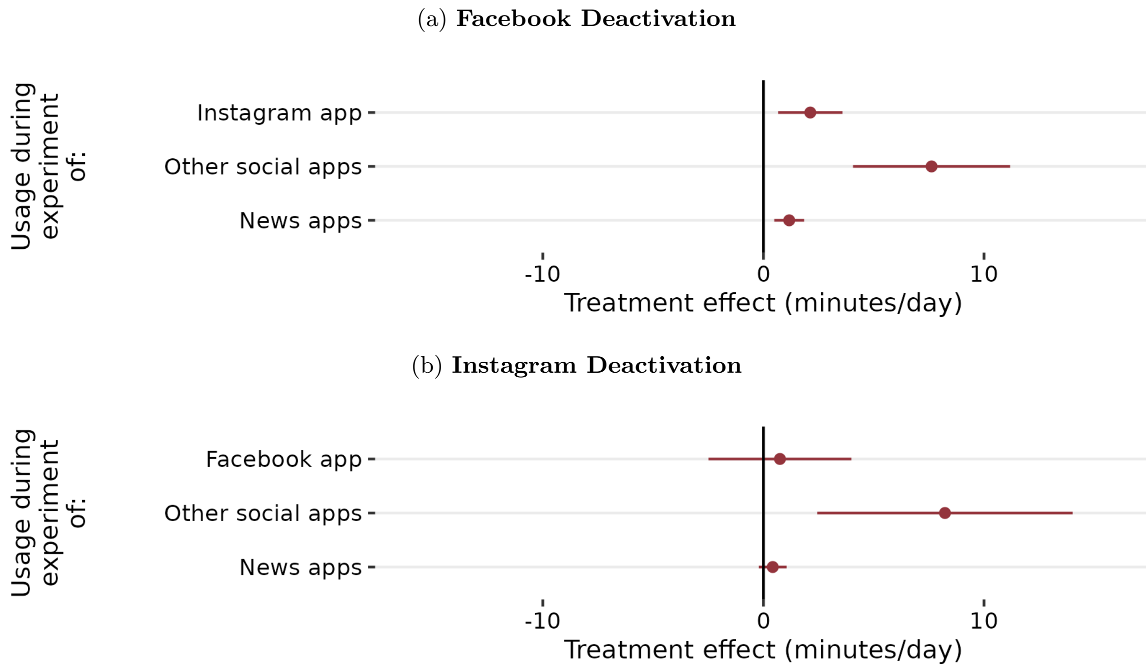
Notes: This figure presents a heat map describing the joint distribution of Facebook and Instagram time use in the baseline period (April 12–May 2, 2023) of the Digital Addiction experiment (Allcott, Gentzkow, and Song, 2022). The histograms at the top and right present the marginal distributions of Facebook and Instagram use, respectively. Single-homers (consumers with exactly zero time use on a platform) are plotted separately as values less than zero.

Figure 3: **Effects of Deactivation on Focal Platform Use**



Notes: This figure presents the share of Deactivation and Control groups that used Facebook and Instagram on each day of the 2020 Facebook and Instagram Election Study. "Use" is defined as logging in and seeing five or more pieces of content. The dark grey shaded area indicates the Control group's 7-day deactivation period, while the light grey shaded area indicates the Deactivation group's 35-day additional deactivation period. We exclude Facebook use data from October 27th due to a logging error. *This figure and figure note are from* (Allcott et al., 2024). *When we receive access to the FIES data, we will create new figures that are directly tailored to our analysis.*

Figure 4: **Effects of Deactivation on Substitute App Use**

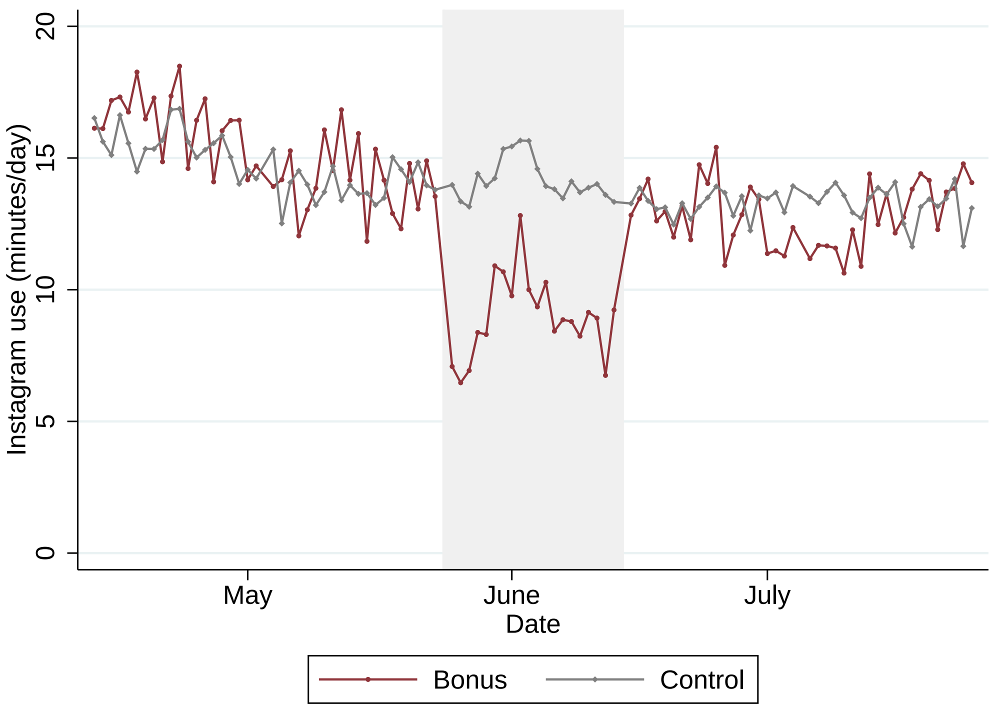


Notes: Panels (a) and (b) present local average treatment effects of Facebook and Instagram deactivation in the 2020 Facebook and Instagram Election Study passive tracking sample. *These figures are from Allcott et al. (2023). When we receive access to the FIES data, we will create new figures that are directly tailored to our analysis.*

Figure 5: **Digital Addiction Facebook and Instagram Use**

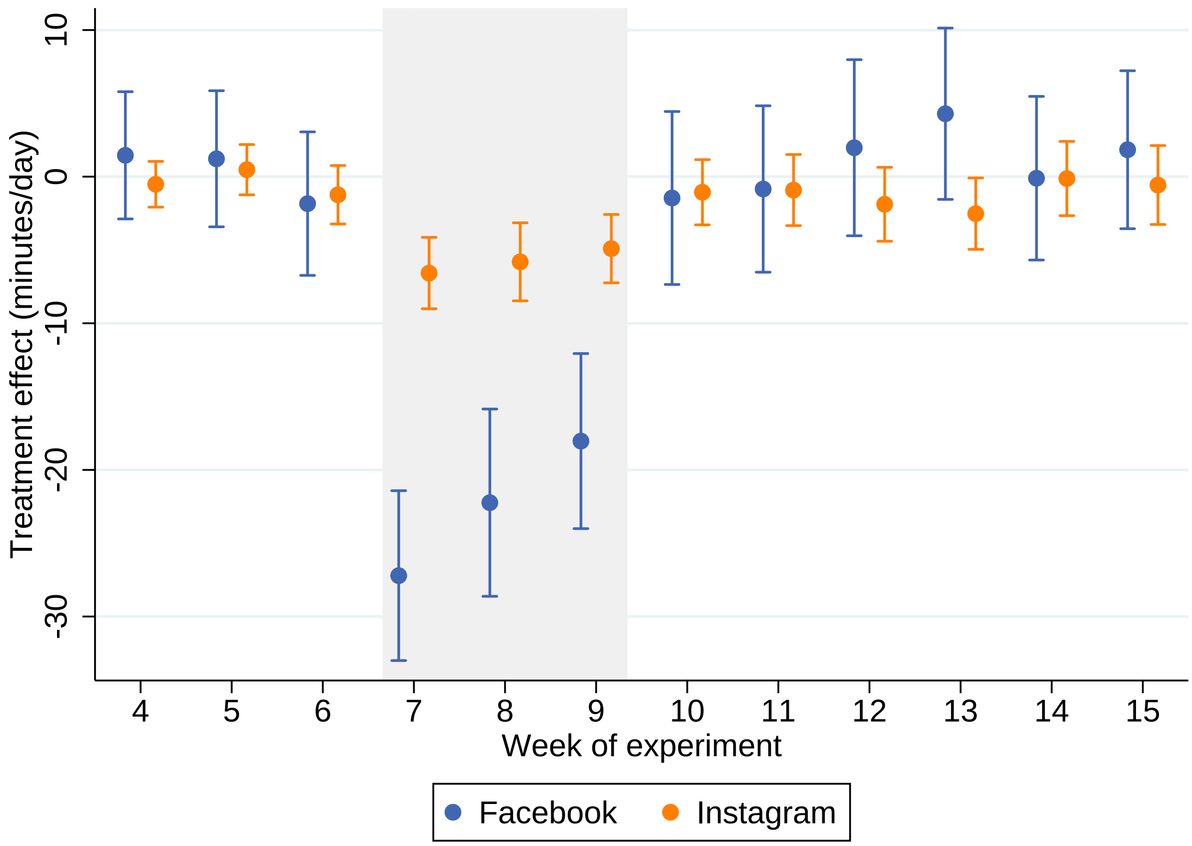
(a) **Facebook Use**

(b) **Instagram Use**



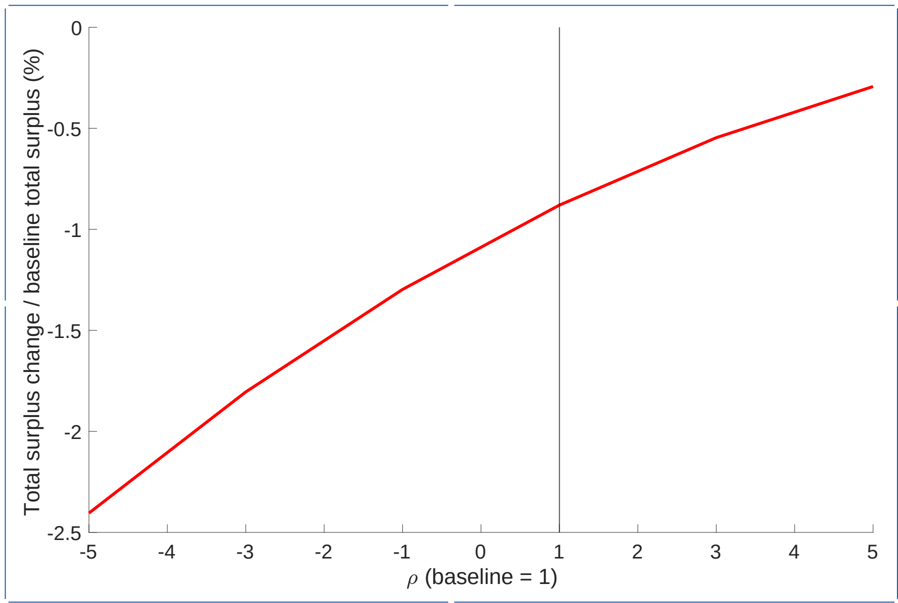
Notes: Panels (a) and (b) present average Facebook and Instagram use in the Digital Addiction experiment Bonus and Bonus Control groups, limiting the sample to the Limit Control group.

Figure 6: **Effects of Screen Time Bonus on Facebook and Instagram Use**

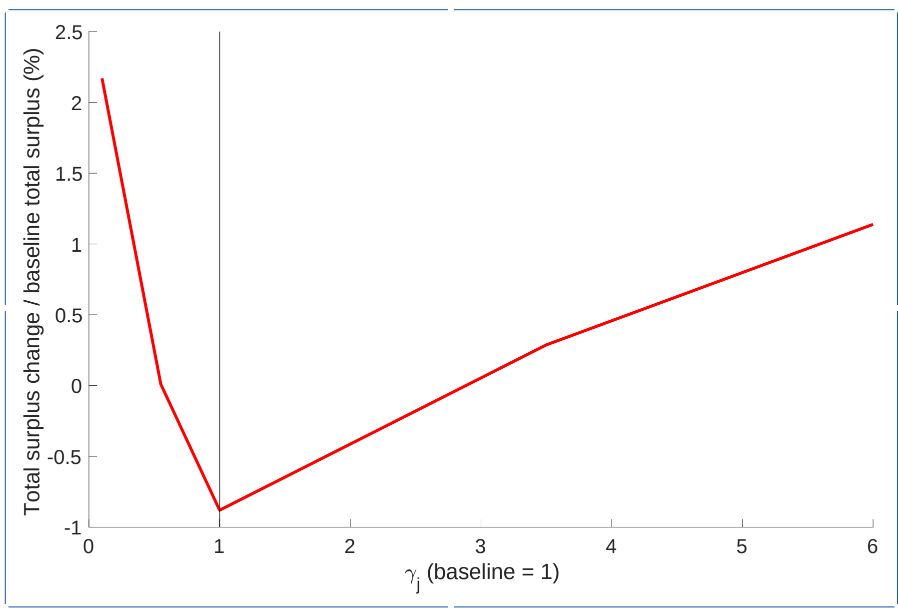


Notes: This figure presents the effects of the Digital Addiction experiment Screen Time Bonus on Facebook and Instagram use. The grey shaded region indicates the 20-day period when the Bonus group was being paid $2.50 per hour to reduce use of Facebook, Instagram, Twitter, Snapchat, web browsers, and YouTube.

Figure 7: **Counterfactual Total Surplus Sensitivity: User-Side Parameters**



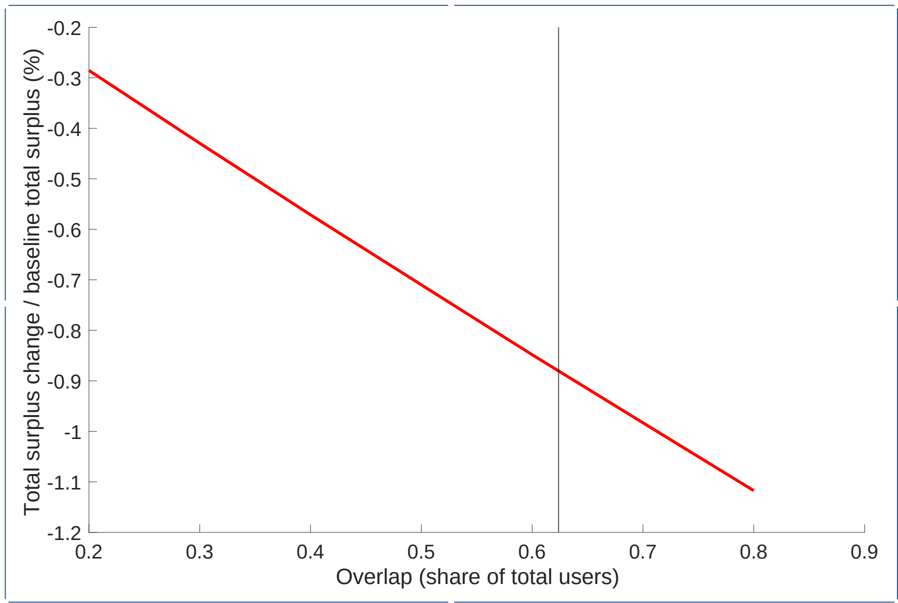
(a) **Platform Substitution**



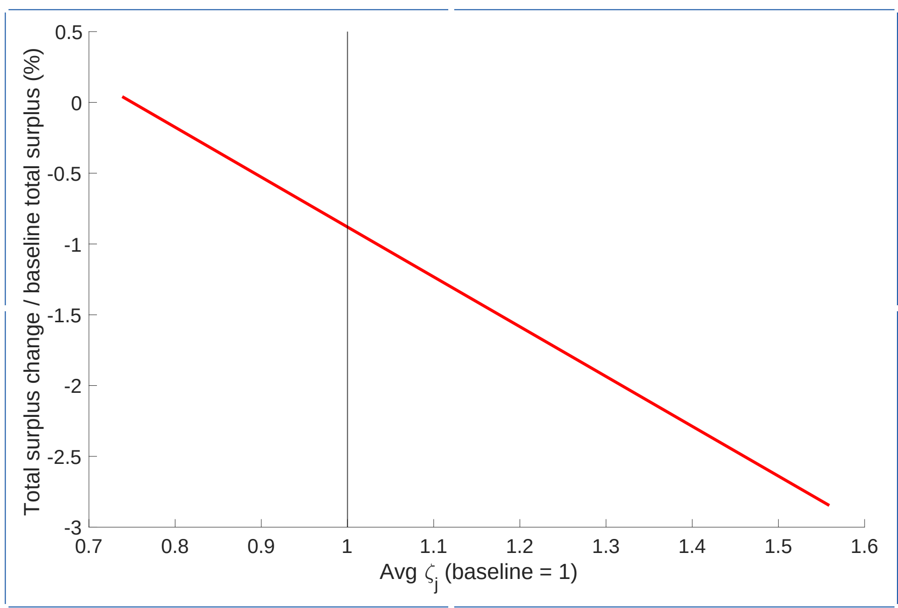
(b) **Ad Disutility**

Notes: This figure presents the impact of alternative user-side parameters on estimates of the total surplus effects of a Facebook-Instagram separation. We perturb the parameter on the horizontal axis, compute ad load in the new merged and separated equilibria, and calculate the change in total surplus from the merged to separated equilibrium as a fraction of the total surplus in the merged equilibrium with baseline parameter values. Higher ρ indicates that platforms are stronger substitutes, and higher average γ_j implies that users are more averse to ad load.

Figure 8: **Counterfactual Total Surplus Sensitivity**: **User-Side Parameters**



(a) **Overlap**



(b) **Loss from Duplicated Impressions**

Notes: This figure presents the impact of alternative advertiser-side parameters on estimates of the total surplus effects of a Facebook-Instagram separation. We perturb the parameter on the horizontal axis, compute ad load in the new merged and separated equilibria, and calculate the change in total surplus from the merged to separated equilibrium as a fraction of the total surplus in the merged equilibrium with baseline parameter values. Higher overlap maintains the ratio of single-homers on Facebook to Instagram, and varies the total number of multi-homers as a fraction of total users. Higher average $\zeta_j$indicates a greater loss from duplciation, with the range of plotted ζ_j generated from $\kappa \in [0, 1]$.

Online Appendix

Digital Media Mergers: Theory and Application to Facebook-Instagram

Justin Katz and Hunt Allcott

Table of Contents

A Model Appendix

A.1 Alternative to Constant Impressions Per Campaign

This subsection provides an alternative assumption that delivers equivalent equilibrium prices as the assumption in the text that users have the same optimal impressions per campaign m.

Assumption 3. *The optimal number of impressions per campaign for user i and advertiser a is given by:*

$$m_{ia}(\omega_{ia}\pi_a, p_i) = \frac{f \cdot \omega_{ia}\pi_a}{p_i + \eta \cdot (1+\eta_0)}$$

where f is a constant.

Assumption 3 has sensible comparative statics – the optimal number of impressions is increasing in profits per impression, and decreasing in price per impression. The specific functional form ensures that market clearing prices are unchanged relative to their implied value if $f = 2m$. Under Assumption 3, the market clearing condition in the merged equilibrium becomes:

$$\begin{aligned}
\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) &= \sum_a m_i \cdot \mathbf{1}[p_i \leq \omega_{ia}\pi_a] \\
&= E[m_i | \omega_{ia}\pi_a \geq p_i] \cdot A \cdot (1 - H_i(p_i)) \\
&= \frac{f}{2} \cdot \frac{1}{p_i + \eta \cdot (1+\eta_0)} \cdot (p_i + \eta \cdot (1+\eta_0))\, A \cdot (1 - H_i(p_i)) \\
&= m \cdot A \cdot (1 - H_i(p_i)) \qquad (40)
\end{aligned}$$

where equation (40) is the same as equation (3) under the assumption in the main text.

A.2 Derivation of Marginal Overlap Function

This subsection derives the marignal overlap function in Section 1.2.2.

Proof. By definition:

$$\begin{aligned}
O'_{aj}(\boldsymbol{q}) &= \Pr(i \text{ (infra)marginal on } -j \text{ given } q_{-j} \text{ given } i \text{ marginal on } j \text{ given } q_j) \\
&= \Pr\left(\frac{p_{i,-j}}{\omega_{ia}} \leq c_{a,-j}(q_{-j}) | \frac{p_{ij}}{\omega_{ia}} = c_{aj}(q_j)\right) \\
&= \Pr\left(\frac{p_{i,-j}}{p_{ij}} \leq \frac{c_{a,-j}(q_{-j})}{c_{aj}(q_j)}\right) \\
&= \Pr\left(\frac{p_{i,-j}}{p_{ij}} \leq \frac{\pi_a\left(1 - \zeta_{-j} O'_{a,-j}(\boldsymbol{q})\right)}{\pi_a\left(1 - \zeta_j O'_{aj}(\boldsymbol{q})\right)}\right) \qquad (41)
\end{aligned}$$

where the last line applies the fact that $c_{aj}(q_j) = \pi_a\left(1 - \zeta O'_{aj}\right)$.

Next, use market clearing to get $\frac{p_{i,-j}}{p_{ij}}$:

$$\begin{aligned}\alpha_j T_{ij}(\boldsymbol{\alpha}) &= \sum_a \mathbf{1}\left[\pi_a\left(1-\zeta_j O'_{aj}(\boldsymbol{q})\right)\omega_{ia} \geq p_{ij}\right] \\ &= A\left[1 - H_i\left(\frac{p_{ij}}{1-\zeta_j O'_{aj}(\boldsymbol{q})}\right)\right]\end{aligned}$$

Going from the first line to the second implicitly applies Assumption 1 because it says that the distribution of $\pi_a \omega_{ia}$ does not depend on (for instance) different time usage on different platforms and can be described by a single person-specific distribution. This follows from independent click-through rates. Solving for p_{ij}:

$$p_{ij} = \left(1-\zeta_j O'_{aj}(\boldsymbol{q})\right) H_i^{-1}\left(1 - \frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{Am}\right)$$

Therefore:

$$\frac{p_{i,-j}}{p_{ij}} = \frac{\left(1-\zeta_{-j} O'_{a,-j}(\boldsymbol{q})\right) H_i^{-1}\left(1 - \frac{\alpha_{-j} T_{i,-j}(\boldsymbol{\alpha})}{Am}\right)}{\left(1-\zeta_j O'_{aj}(\boldsymbol{q})\right) H_i^{-1}\left(1 - \frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{Am}\right)} \tag{42}$$

Substituting (42) into (41), the ratios $\frac{\left(1-\zeta_{-j} O'_{a,-j}(\boldsymbol{q})\right)}{\left(1-\zeta_j O'_{aj}(\boldsymbol{q})\right)}$ cancel, and we end up with:

$$\begin{aligned}O'_{aj}(\boldsymbol{q}^*) = O'_{aj} &= \Pr\left(\frac{H_i^{-1}\left(1 - \frac{\alpha_{-j} T_{i,-j}(\boldsymbol{\alpha})}{Am}\right)}{H_i^{-1}\left(1 - \frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{Am}\right)} \leq 1\right) \\ &= \Pr[\alpha_j T_{ij}(\boldsymbol{\alpha}) \leq \alpha_{-j} T_{i,-j}(\boldsymbol{\alpha})] \\ &= \frac{\sum_{i \in \mathcal{U}_j} \mathbf{1}[\alpha_j T_{ij}(\boldsymbol{\alpha}) \leq \alpha_{-j} T_{i,-j}(\boldsymbol{\alpha})]}{N_j}\end{aligned}$$

Going from the first to the second line to the second uses the fact that H_i^{-1} is monotone increasing. The third line follows by definition and gives the expression in the text. □

A.3 Presentation of Homogeneous Users, Constant Click-Through Rate, and No Duplication Special Case

See Appendix A.4 for derivations.

A.3.1 Social Optimum

As a benchmark, we consider a constrained social planner who sets ad load and advertiser prices to maximize the sum of user and advertiser surplus, subject to user optimization and merged equilibrium market clearing constraints in equation (3). The social planner chooses:

$$\boldsymbol{\alpha}^{sp} = \arg\max_{\boldsymbol{\alpha}} \sum_i U_i^* \left(\boldsymbol{T}(\boldsymbol{\alpha}), n; \boldsymbol{\alpha}\right) + Am \cdot \int_{x=p(\boldsymbol{\alpha})}^{(\pi\omega)^{\max}} x dH(x). \tag{43}$$

where $U^*(\cdot; \boldsymbol{\alpha}) = \max_{\boldsymbol{T}} U(\cdot; \boldsymbol{\alpha})$. The planner allocates ad slots to users with highest click-through rates, an allocation mimicked by the platform when setting its contract with advertisers.

Define $\gamma_j \equiv -\frac{1}{T_j}\frac{\partial U^*}{\partial \alpha_j} > 0$ as the marginal disutility from ads given optimized time use. The planner solution satisfies

$$\alpha_j^{sp} = \frac{\overbrace{\gamma_j T_j(\boldsymbol{\alpha})}^{\text{consumer surplus}} \quad \overbrace{- p(\boldsymbol{\alpha})\left(T_j(\boldsymbol{\alpha}) + \alpha_{-j}^{sp}\frac{\partial T_{-j}}{\partial \alpha_j}(\boldsymbol{\alpha})\right)}^{\text{advertiser surplus}}}{p(\boldsymbol{\alpha}) \cdot \frac{\partial T_j}{\partial \alpha_j}(\boldsymbol{\alpha})}, \quad j = 1, 2. \tag{44}$$

where the denominator is negative. The expression says that the social planner sets ad load to equalize the marginal consumer surplus loss from additional ads to the marginal advertiser surplus net gain from additional ads. Optimal ad load is high when users' disutility of ads γ is low or advertisers' profits from increased ad loads are high. The value of ad load to advertisers depends on both the equilibrium ad price $p(\boldsymbol{\alpha})$, which signals scarcity of user attention; the equilibrium number of ads served, controlled by $T_j(\boldsymbol{\alpha})$; and the impact of ad load on user time on platform, controlled by $\frac{\partial T_{-j}}{\partial \alpha_j}$ and $\frac{\partial T_j}{\partial \alpha_j}$. Ad load on platform j is higher when $\frac{\partial T_{-j}}{\partial \alpha_j}$ is high, i.e. the platforms are substitutes. In that case, some of the lost time use on j due to higher α_j increases time use, and hence ad impressions, on $-j$, rather than resulting in lost impressions.

A.3.2 Merged Equilibrium

The merged platform chooses

$$\boldsymbol{\alpha}^{\boldsymbol{m}} = \arg\max_{\boldsymbol{\alpha}} \; p(\boldsymbol{\alpha}) \cdot \sum_i \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}). \tag{45}$$

The solution satisfies

$$\alpha_j^m = \frac{\overbrace{-\frac{\partial p(\boldsymbol{\alpha})}{\partial \alpha_j} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}(\boldsymbol{\alpha})}^{\text{infra-marginal revenue}} \overbrace{- p(\boldsymbol{\alpha}) \cdot \left(T_j(\boldsymbol{\alpha}) + \alpha_{-j}\frac{\partial T_{-j}}{\partial \alpha_j}(\boldsymbol{\alpha})\right)}^{\text{marginal revenue}}}{p(\boldsymbol{\alpha}) \cdot \frac{\partial T_j}{\partial \alpha_j}(\boldsymbol{\alpha})}, \quad j = 1, 2. \tag{46}$$

The expression says that the merged platform sets ad load to equalize the marginal revenue loss from infra-marginal impressions and the net marginal revenue impact from marginal impressions due to high ad load. Higher ad load reduces revenue on infra-marginal impressions by lowering equilibrium prices, but has an ambiguous effect on revenue from marginal impressions. The effect on revenue from marginal impressions is increasing in a direct effect from additional ad load on infra-marginal time use and decreasing in the endogenous indirect effect of lower time use due to higher ad load.

The merged platform solution aligns with the planner solution if

$$-\frac{\partial p(\boldsymbol{\alpha}^{sp})}{\partial \alpha_j} \cdot \boldsymbol{\alpha}^{sp} \cdot \boldsymbol{T}(\boldsymbol{\alpha}^{sp}) = \gamma_j T_j(\boldsymbol{\alpha}^{sp}), \quad j = \{1, 2\}\,. \tag{47}$$

where the right-hand side of Equation (47) equals the total utility loss on platform j due to higher ad load. This highlights the distortion in the merged equilibrium: the platform holds back ad load to increase equilibrium prices, rather than to avoid consumer harm. This disconnect means there is no reason to think that equation (47) will hold, or that the merged equilibrium will tend to produce ad load that is higher or lower than socially optimal—an ambiguity that is typical in the two-sided markets literature. Roughly, as price becomes more sensitive to ad load, so that $-\frac{\partial p}{\partial \alpha_j}$ increases, then ad load in a merged competitive equilibrium is likely be to low, and vice versa as price becomes less sensitive to ad load.

A.3.3 Separated Equilibrium

Suppose that separated platforms set α_j simultaneously, taking as given their rival's actions. Each separated platform solves

$$\alpha_j^S = \arg\max_{\alpha_j}\ p(\alpha_j, \alpha_{-j}) \cdot \alpha_j \cdot \sum_{i \in \mathcal{U}_j} T_{ij}(\alpha_j, \alpha_{-j}). \tag{48}$$

Price $p(\boldsymbol{\alpha})$ is still given by by Equation (4) because platforms coordinate to avoid duplication. The solution satisfies

$$\alpha_j^S = \frac{-\frac{\partial p(\boldsymbol{\alpha})}{\partial \alpha_j} \cdot \alpha_j \cdot T_j(\boldsymbol{\alpha}) - p(\boldsymbol{\alpha}) \cdot T_j(\boldsymbol{\alpha})}{p(\boldsymbol{\alpha}) \cdot \frac{\partial T_j}{\partial \alpha_j}(\boldsymbol{\alpha})} \quad j = 1, 2. \tag{49}$$

This differs from equation (46) in two ways. First, separated platforms ignore the impact of increased ad load on revenue from infra-marginal impressions on platform $-j$. This is a standard Cournot externality that increases ad load relative to the merged equilibrium. Second, separated platforms do not consider how ad load on j will indirectly impact time use on $-j$ through user substitution, reflected in the missing $\alpha_{-j}\frac{\partial T_{-j}}{\partial \alpha_j}$ term. When platforms are substitutes, i.e. $\frac{\partial T_{-j}}{\partial \alpha_j} > 0$, the merged platform loses less revenue from lost marginal impressions than separated platforms because consumers substiute to the other platform. This time use diversion tends to lower α_j^S relative to α_j^m. The argument is reversed if platforms are complements, i..e $\frac{\partial T_{-j}}{\partial \alpha_j} < 0$.

Overall, ad load in the separated equilibrium may be higher or lower than in the merged equilibrium. If platforms are very strong substitutes, time use diversion may overwhelm the Cournot externality and decrease ad load when platforms separate. Separating platforms may therefore bring ad load closer to or further from the social optimum. Reducing market power does not unambiguously reduce the distortion in competitive equilibrium. This is because the distortion is not due to platform pricing power, but because user time use is not directly priced, so user utility does not directly enter the platform problem.

Summary. In this simplified environment, the merged platform equilibrium may have higher or lower ad load relative to the socially optimal level. Separating platform ownership may increase or decrease ad load relative to when platforms are merged, and hence might mitigate or exacerbate the monopoly distortion. The impact of platform separation on ad load in competitive equilibrium depends on user-side diversion, parameterized by $\frac{\partial T_{-j}}{\partial \alpha_j}$.

A.4 Derivations for Section 1.3

This subsection derives expressions in Section 1.3.

Derivation of Equation (44). First, note that given assumptions in Section 1.3:

$$\sum_i U_i^* \left(\boldsymbol{T}(\boldsymbol{\alpha}), n; \boldsymbol{\alpha}\right) = NU^* \left(\boldsymbol{T}(\boldsymbol{\alpha}), n; \boldsymbol{\alpha}\right);$$

The first-order conditions for Problem (43) are therefore:

$$\begin{aligned} 0 &= \frac{\partial}{\partial \alpha_j}\left[NU^*\left(\boldsymbol{T}(\boldsymbol{\alpha}), n; \boldsymbol{\alpha}\right) + NAm \cdot \int_{x=p(\boldsymbol{\alpha})}^{(\pi\omega)^{\max}} x dH(x)\right] \\ &= N\frac{\partial U^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} - NAm \cdot p\left(\boldsymbol{\alpha}\right) \cdot h\left(p(\boldsymbol{\alpha})\right) \cdot \frac{\partial p\left(\boldsymbol{\alpha}\right)}{\partial \alpha_j} \end{aligned}$$

Note that from Equation (4), applying the inverse function rule:

$$\frac{\partial p\left(\boldsymbol{\alpha}\right)}{\partial \alpha_j} = -\left(Am \cdot h\left(p\left(\boldsymbol{\alpha}\right)\right)\right)^{-1} \cdot \left[T_j + \alpha_j \frac{\partial T_j}{\partial \alpha_j} + \alpha_{-j}\frac{\partial T_{-j}}{\partial \alpha_j}\right] \tag{50}$$

The FOC becomes:

$$0 = \frac{\partial U^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} + p\left(\boldsymbol{\alpha}\right) \cdot \left[T_j + \alpha_j \frac{\partial T_j}{\partial \alpha_j} + \alpha_{-j}\frac{\partial T_{-j}}{\partial \alpha_j}\right]$$

Solving for α_j:

$$\alpha_j = \frac{-\frac{\partial U^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} - p(\boldsymbol{\alpha}) \cdot \left(T_j + \alpha_{-j}\frac{\partial T_{-j}}{\partial \alpha_j}\right)}{p(\boldsymbol{\alpha}) \cdot \frac{\partial T_j}{\partial \alpha_j}}$$

which gives the expression in the text, after substituting $\gamma_j = \frac{1}{T_j}\frac{\partial U^*}{\partial \alpha_j}$. □

Derivation of Equation (46). The first-order conditions for Problem (45) are:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial \alpha_j}\left[p(\boldsymbol{\alpha}) \cdot N \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}(\boldsymbol{\alpha})\right] \\
&= \frac{\partial p}{\partial \alpha_j} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T(\alpha)} + p(\boldsymbol{\alpha}) \cdot T_j(\boldsymbol{\alpha}) + p(\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}(\boldsymbol{\alpha})}{\partial \alpha_j}
\end{aligned}
$$

where $\frac{\partial \boldsymbol{T}(\boldsymbol{\alpha})}{\partial \alpha_j}$ is a Jacboian. Expanding out the vector $\boldsymbol{\alpha}$ in the third term and rearranging yields:

$$
\alpha_j = \frac{-\frac{\partial p(\boldsymbol{\alpha})}{\partial \alpha_j} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}(\boldsymbol{\alpha}) - p(\boldsymbol{\alpha}) \cdot \left(T_j(\boldsymbol{\alpha}) + \alpha_{-j} \frac{\partial T_{-j}}{\partial \alpha_j}(\boldsymbol{\alpha})\right)}{p(\boldsymbol{\alpha}) \cdot \frac{\partial T_j}{\partial \alpha_j}(\boldsymbol{\alpha})}
$$

which gives the expression in the text. □

Derivation of Equation (49) . The first-order conditions for Problem (48) are:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial \alpha_j}\left(p(\alpha_j, \alpha_{-j}) \cdot \alpha_j \cdot N_j \cdot T_j(\alpha_j, \alpha_{-j})\right) \\
&= \frac{\partial p}{\partial \alpha_j} \cdot \alpha_j \cdot T_j(\boldsymbol{\alpha}) + p(\boldsymbol{\alpha}) \cdot T_j + p(\boldsymbol{\alpha}) \cdot \alpha_j \cdot \frac{\partial T_j}{\partial \alpha_j}(\boldsymbol{\alpha})
\end{aligned}
$$

Solving for α_j yields:

$$
\alpha_j = \frac{-\frac{\partial p}{\partial \alpha_j} \cdot \alpha_j \cdot T_j(\boldsymbol{\alpha}) - p(\boldsymbol{\alpha}) \cdot T_j}{p(\boldsymbol{\alpha}) \cdot \alpha_j \cdot \frac{\partial T_j}{\partial \alpha_j}(\boldsymbol{\alpha})}
$$

which gives the expression in the text. □

A.5 Derivations for Section 1.4

This subsection derives expressions in Section 1.4.

Derivation of Equation (12). The monopolist problem is:

$$
\boldsymbol{\alpha}^{e,m} = \arg\max_{\boldsymbol{\alpha}} \sum_i p_i \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i
$$

where the only difference relative to Equation (5) is that time use does not depend on $\boldsymbol{\alpha}$ as it is exogenous. The first-order conditions are:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial \alpha_j}\left[\sum_i p_i \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i\right] \\
&= \sum_i \boldsymbol{\alpha} \cdot \boldsymbol{T}_i \cdot \frac{\partial p_i}{\partial \alpha_j} + p_i \cdot T_{ij}
\end{aligned}
$$

Rearranging, we get the expression in the text:

$$\alpha_j = -\frac{\sum_i \alpha_{-j} \cdot T_{i,-j} \cdot \frac{\partial p_i}{\partial \alpha_j} + T_{ij} \cdot p_i}{\sum_i T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}} \quad \text{where } \frac{\partial p_i}{\partial \alpha_j} = -\left(Am \cdot h_i(p_i(\boldsymbol{\alpha}))\right)^{-1} \cdot T_{ij}$$

where the value of $\frac{\partial p_i}{\partial \alpha_j}$ comes from differentiating Equation (4) with respect to α_j and applying the inverse function rule. □

Derivation of Equation (14), no duplication. The separated platform problem is:

$$\alpha_j^{e,s,i} = \arg\max_{\alpha_j} \sum_{i \in \mathcal{U}_j} \alpha_j \cdot T_{ij} \cdot p_i \tag{51}$$

The first-order condition is:

$$0 = \sum_{i \in \mathcal{U}_j} \alpha_j \cdot T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j} + T_{ij} \cdot p_i$$

where $\frac{\partial p_i}{\partial \alpha_j}$ is as in Equation (12). Rearranging gives the expression in the main text:

$$\alpha_j = -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_i}{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}}$$

□

Derivation of Equation (14), with duplication. The first-order condition for Problem (13) is:

$$0 = \sum_{i \in \mathcal{U}_j} \alpha_j \cdot T_{ij} \cdot \frac{\partial p_{ij}}{\partial \alpha_j} + T_{ij} \cdot p_{ij}$$

Rearranging:

$$\alpha_j = -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_{ij}}{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}}$$

where:

$$\frac{\partial p_{ij}}{\partial \alpha_j} = -\left(1 - O'_{aj}\right) \cdot \left(Am \cdot h_i(p_{ij})\right)^{-1} \cdot T_{ij} - \frac{p_{ij}}{\left(1 - O'_{aj}\right)} \frac{\partial O'_{aj}}{\partial \alpha_j}$$

which gives the expressions in the text. □

Proposition 1 (Advertiser side strategic complementarity)**.** *Suppose that time use $\boldsymbol{T}_i$ is exogenous with $(T_{i1}, T_{i2}) \sim \mathcal{T}$ such that $\frac{T_{ij}}{T_{i,-j}}$ has a well-defined distribution, N_j represents the continuous mass of users on platform j so that O'_{aj} is differentiable, and Assumptions 1 and 2 hold. Then ad load choices are strategic substitutes in the separated platform solution with no duplication described in Problem (51) and either strategic complements or substitutes in the separarted platform solution with duplication described in Problem (13).*

Proof. To show ad load choices are strategic substitutes in Problem (51), first note that given Assumptions Assumptions 1 and 2:

$$\frac{\partial p_i}{\partial \alpha_j} = -\frac{\eta}{Am} \cdot T_{ij} \implies \frac{\partial^2 p_i}{\partial \alpha_{-j} \partial \alpha_j} = 0$$

Differentiate Equation (14) with respect to α_{-j} in the case of no duplciation, and substitute $p_{ij} = p_i$:

$$\begin{aligned}\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} &= -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_{-j}}}{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_i}{\partial \alpha_j}} \\ &= -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} T_{i,-j}}{\sum_{i \in \mathcal{U}_j} T_{ij}^2}\end{aligned}$$

This proves that $\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} \leq 0$, meaning choices of ad load are strategic substitutes.

To show that ad load choices are strategic complements in Problem (13), define the distribution of $T_{ij}/T_{i,-j}$ as $\mathcal{T}_{-j}$ and note that:

$$O'_{aj} = \Pr\left(\alpha_j T_{ij} \leq \alpha_{-j} T_{i,-j}\right) = \mathcal{T}_{-j}\left(\alpha_{-j}/\alpha_j\right)$$

Therefore:

$$\frac{\partial O'_{aj}}{\partial \alpha_{-j}} = \mathcal{T}'_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right) \cdot \alpha_j^{-1} \geq 0, \quad \frac{\partial O'_{aj}}{\partial \alpha_j} = -\mathcal{T}'_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right) \cdot \frac{\alpha_{-j}}{\alpha_j^2} \leq 0$$

Moreover:

$$\frac{\partial O'_{aj}}{\partial \alpha_{-j} \partial \alpha_j} = - - \mathcal{T}'_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right) \cdot \frac{1}{\alpha_j^2} - \frac{\alpha_{-j}}{\alpha_j^3} \cdot \mathcal{T}''_{-j}\left(\frac{\alpha_{-j}}{\alpha_j}\right)$$

which has ambiguous sign. Differentiate Equation (14) with respect to α_{-j} in the case with duplication:

$$\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} = -\frac{\sum_{i \in \mathcal{U}_j} T_{ij} \cdot p_{ij}}{\left(\sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial p_{ij}}{\partial \alpha_j}\right)^2} \cdot \sum_{i \in \mathcal{U}_j} T_{ij} \cdot \frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j}$$

Moreover:

$$\begin{aligned}\frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j} = &\zeta \frac{\partial O'_{aj}}{\partial \alpha_{-j}} \cdot \frac{\eta}{Am} T_{ij} + \zeta \frac{p_{ij}}{\left(1 - O'_{aj}\right)^2} \frac{\partial O'_{aj}}{\partial \alpha_{-j}} \frac{\partial O'_{aj}}{\partial \alpha_j} \\ &- \frac{p_{ij}}{\left(1 - O'_{aj}(\boldsymbol{\alpha}^{e,s,d}\right)} \frac{\partial^2 O'_{aj}}{\partial \alpha_{-j} \partial \alpha_j}\end{aligned}$$

The first term is positive because $\frac{\partial O'_{aj}}{\partial \alpha_{-j}}$ is. The second term is negative because $\frac{\partial O'_{aj}}{\partial \alpha_j}$ is. The third term has a sign depending on the sign of $\frac{\partial^2 O'_{aj}}{\partial \alpha_{-j} \partial \alpha_j}$, which is ambiguous and depends on the shape of the pdf of $T_{ij}/T_{i,-j}$. Overall, it is possible that $\frac{\partial^2 p_{ij}}{\partial \alpha_{-j} \partial \alpha_j} < 0$, implying that $\frac{\partial \alpha_j^{e,s,i}}{\partial \alpha_{-j}} < 0$ (making ad load choices strategic complements). □

Remark. Proposition 1 holds under the weaker assumption that H_i is linear but not necessarily identical across users.

A.5.1 Derivations for Section 1.4.3

This subsection derives expressions in Section 1.4.3.

Derivation of monopoly ad load. Monopoly revenue is:

$$R^m(\boldsymbol{\alpha}) = \eta \cdot \left(\frac{O}{2}\left(\alpha_1 + \alpha_2\right) \cdot \left(1 + \eta_0 - \frac{\alpha_1 + \alpha_2}{2Am}\right) + \sum_j (N_j - O)\alpha_j \left(1 + \eta_0 - \frac{\alpha_j}{Am}\right)\right)$$

The first-order condition with respect to α_j is:

$$\begin{aligned} 0 &= \frac{O}{2}\left(\left(1 + \eta_0 - \frac{\alpha_j + \alpha_{-j}}{Am}\right) + (N_j - O)\left(1 + \eta_0 - 2\frac{\alpha_j}{Am}\right)\right) \\ &= \frac{\alpha_j}{Am}\left(3O - 4N_j\right) + \left(1 + \eta_0\right)\left(2N_j - O\right) - \frac{O}{A}\alpha_{-j} \end{aligned}$$

Therefore:

$$\alpha_j = \frac{2N_j - O}{4N_j - 3O} Am(1 + \eta_0) - \frac{O}{4N_j - 3O}\alpha_{-j}$$

Substituting in for α_{-j} and simplifying gives:

$$\alpha_j = \left(\frac{\left(2N_j - O\right)\left(4N_{-j} - 3O\right) - O(2N_{-j} - O)}{\left(4N_j - 3O\right) \cdot \left(4N_{-j} - 3O\right) - O^2}\right) Am(1 + \eta_0)$$

The numerator simplifies to $8N_jN_{-j} - 6O(N_j + N_{-j}) + 4O^2$ and the denominator simplifies to $2 \cdot \left(8N_jN_{-j} - 6O(N_j + N_{-j}) + 4O^2\right)$. Therefore, the expression becomes:

$$\alpha_j^{e,m} = \frac{1}{2}Am(1 + \eta_0) \tag{52}$$

as reported in the text. □

Derivation of Equation (16). Separated platform revenue is:

$$R_j^s(\boldsymbol{\alpha}) = \eta \cdot \left(\frac{O}{2}\alpha_j \cdot \left(1 + \eta_0 - \frac{\alpha_1 + \alpha_2}{2Am}\right) + (N_j - O)\alpha_j \left(1 + \eta_0 - \frac{\alpha_j}{Am}\right)\right)$$

The first-order condition is:

$$\begin{aligned} 0 &= \frac{O}{2}\left(1 + \eta_0 - \frac{\alpha_j + \alpha_{-j}}{2Am} - \frac{\alpha_j}{2Am}\right) + (N_j - O)\left(1 + \eta_0 - 2\frac{\alpha_j}{Am}\right) \\ &= \frac{\alpha_j}{Am}\left(3O - 4N_j\right) + \left(1 + \eta_0\right)\left(2N_j - O\right) - \frac{O}{Am}\alpha_{-j} \end{aligned}$$

Therefore:

$$\alpha_j^{e,s,i}(\alpha_{-j}) = \frac{2N_j - O}{4N_j - 3O} Am(1+\eta_0) - \frac{O}{(4N_j - 3O)}\alpha_{-j}$$
$$= \frac{2-\mu_j}{4-3\mu_j} Am(1+\eta_0) - \frac{\mu_j}{4-3\mu_j}\alpha_{-j}$$

where we factor out N_j from the numerator and denominator to express in terms of μ_j. Substituting for α_{-j} and simplifying yields:

$$\alpha_j^{e,s,i} = \left(\frac{(2N_j - O)(4N_{-j} - 3O) - O(2N_{-j} - O)}{(4N_j - 3O)\cdot(4N_{-j} - 3O) - O^2} \right) Am(1+\eta_0) \tag{53}$$

□

Percent change in ad load only depends on overlap statistics. Using Equations (52) and (53), the percent increase in ad load in the separated equilibrium is:

$$\frac{\alpha_j^{e,s,i}}{\alpha_j^{e,m}} - 1 = 2\cdot\left(\frac{(2N_j - O)(4N_{-j} - 3O) - O(2N_{-j} - O)}{(4N_j - 3O)\cdot(4N_{-j} - 3O) - O^2} \right) - 1$$
$$= 2\cdot\left(\frac{(2-\mu_j)(4-3\mu_{-j}) - \mu_j(2-\mu_{-j})}{(4-3\mu_j)\cdot(4-3\mu_{-j}) - \mu_j\mu_{-j}} \right) - 1$$

This only depends on overlap statistics, proving the claim in the text. □

A.6 Full equilibrium analysis for Section 1.5

This section analyzes the full model under various ownership structures. Throughout, we will apply assumptions 1 and 2.

A.6.1 Social planner benchmark

The analysis follows Section A.3.1 pointwise. The planner chooses:

$$\boldsymbol{\alpha}^{sp} = \arg\max_{\boldsymbol{\alpha}} \sum_i U_i^*\left(\boldsymbol{T}(\boldsymbol{\alpha}), n; \boldsymbol{\alpha}\right) + \sum_i Am \cdot \int_{x=p_i(\boldsymbol{\alpha})}^{(\pi\omega)^{\max}} x dH(x)$$

The first-order condition is:

$$0 = \sum_i \frac{\partial U_i^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} - \sum_i Am \cdot p_i(\boldsymbol{\alpha}) \cdot h\left(p_i(\boldsymbol{\alpha})\right) \cdot \frac{\partial p_i(\boldsymbol{\alpha})}{\partial \alpha_j}$$

Since:

$$\frac{\partial p_{ii}(\boldsymbol{\alpha})}{\partial \alpha_j} = -\left(Am \cdot h\left(p_i(\boldsymbol{\alpha})\right)\right)^{-1} \cdot \left[T_{ij} + \alpha_j \frac{\partial T_{ij}}{\partial \alpha_j} + \alpha_{-j}\frac{\partial T_{i,-j}}{\partial \alpha_j} \right]$$

the FOC becomes:

$$0 = \sum_i \frac{\partial U_i^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} - \sum_i p_i(\boldsymbol{\alpha}) \cdot \left[T_{ij} + \alpha_j \frac{\partial T_{ij}}{\partial \alpha_j} + \alpha_{-j} \frac{\partial T_{i,-j}}{\partial \alpha_j} \right]$$

Solving for α_j:

$$\alpha_j = \frac{-\sum_i \frac{\partial U_i^*(\cdot;\boldsymbol{\alpha})}{\partial \alpha_j} - \sum_i p_i(\boldsymbol{\alpha}) \cdot \left(T_{ij} + \alpha_{-j} \frac{\partial T_{i,-j}}{\partial \alpha_j} \right)}{\sum_i p_i(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}} \tag{54}$$

This is conceptually identical to Equation (44) in Section A.3.1 in that the social planner balances the aggregate marginal cost of additional ads on user welfare against the aggregate marginal benefit of additional ads for advertiser welfare. Now, the planner accounts for user heterogeneity when calculating aggregate benefits and costs.

A.6.2 Merged platform solution

The problem is:

$$\max_{\boldsymbol{\alpha}} \sum_i p_i(\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})$$

The first-order conditions are:

$$0 = \sum_i \frac{\partial p_i}{\partial \alpha_j} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) + p_i(\boldsymbol{\alpha}) \cdot T_{ij}(\boldsymbol{\alpha}) + p_i(\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}_i(\boldsymbol{\alpha})}{\partial \alpha_j}$$

Solving for α_j:

$$\alpha_j = \frac{-\sum_i \frac{\partial p_i}{\partial \alpha_j} \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) - \sum_i p_i(\boldsymbol{\alpha}) \cdot \left(T_{ij}(\boldsymbol{\alpha}) - \alpha_{-j} \cdot \frac{\partial T_{i,-j}}{\partial \alpha_j}(\boldsymbol{\alpha}) \right)}{\sum_i p_i(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha})}$$

This is conceptually identical to Equation (46) balancing aggregate infra-marginal revenue gains against aggregate marginal losses, except that aggregates are calculated accounting for variance in user-level time use heterogeneity. As in Section 1.3, ad load may be higher or lower than the social optimum in the separated equilibrium.

A.6.3 Separated platform solution, no duplication.

The problem is:

$$\max_{\alpha_j} \sum_{i \in \mathcal{U}_j} p_i(\boldsymbol{\alpha}) \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha})$$

The first-order conditions are:

$$0 = \sum_i \frac{\partial p_i}{\partial \alpha_j} \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) + p_i(\boldsymbol{\alpha}) \cdot T_{ij} + p_i(\boldsymbol{\alpha}) \cdot \alpha_j \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha})$$

Solving for α_j:

$$\alpha_j = \frac{-\sum_{i\in\mathcal{U}_j} \frac{\partial p_i}{\partial \alpha_j} \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) - \sum_{i\in\mathcal{U}_j} p_i(\boldsymbol{\alpha}) \cdot T_{ij}}{\sum_{i\in\mathcal{U}_j} p_i(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha})}$$

This is similar to Equation (49). However, now the difference in incentives in the separated versus merged equilibrium depend on user overlap. When all users are single-homers, the separated equilibrium is identical to the merged equilibrium. As the share of multi-homers increases, two differences emerge. First, as in Section 1.4.2, the Cournot externality increases, which increases ad load in the separated equilibrium. Second, as in Section A.3.3, if platforms are substitutes, user diversion reduces the incentive to withhold ad load in the merged equilibrium relative to the separated equilibrium, which increases ad load in the merged equilibrium relative to the separated equilibrium. In general, ad load can be higher or lower than in the merged equilibrium, depending on overlap, user diversion, and price elasticity.

A.6.4 Separated platform solution, duplication.

The problem is:

$$\max_{\alpha_j} \sum_{i\in\mathcal{U}_j} p_{ij}(\boldsymbol{\alpha}) \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha})$$

Taking the first-order condition and solving for α_j:

$$\alpha_j = \frac{-\sum_{i\in\mathcal{U}_j} \frac{\partial p_{ij}}{\partial \alpha_j} \cdot \alpha_j \cdot T_{ij}(\boldsymbol{\alpha}) - \sum_{i\in\mathcal{U}_j} p_{ij}(\boldsymbol{\alpha}) \cdot T_{ij}}{\sum_{i\in\mathcal{U}_j} p_{ij}(\boldsymbol{\alpha}) \cdot \frac{\partial T_{ij}}{\partial \alpha_j}(\boldsymbol{\alpha})}$$

where:

$$\frac{\partial p_{ij}}{\partial \alpha_j} = -\left(1 - O'_{aj}\right) \cdot \frac{\eta}{Am} \cdot T_{ij} - \frac{p_{ij}}{\left(1 - O'_{aj}\right)} \frac{\partial O'_{aj}}{\partial \alpha_j}$$

This is similar to Equation (49). However, the Cournot externality represented in the first term of Equation ((49) is replaced by the combination of the inframarginal effect and business stealing effect, both of which increase ad load in the separated equilibrium as described in Section 1.4.2. The absence of $\frac{\partial T_{i,-j}}{\partial \alpha_j}$ reflects the same user-diversion force expressed above. This implies that ad load may be higher or lower than the combined equilibrium social optimum.

However, since the separated equilibrium now has inefficiently duplicated ad impressions, comparing ad load versus the social planner benchmark is not sufficient to make welfare judgements. For example, even if ad load were identical in the separated platform equilibrium with duplication as in Equation (54), social welfare would be lower in the separated equilibrium because some ads would be inefficiently duplicated, lowering advertiser surplus. See Section 5 for details.

B Empirical Appendix

B.1 Duplication loss experiment.

B.1.1 Ad selection and ad design.

We developed 15 creatives for distinct products spanning five product categories to run as ads on Meta platforms in February 2025. We selected products and categories to be representative of typical advertisements on Meta. To do so, we first picked five top product categories and the top three advertisers within each category by ad spending from the 2024 SensorTower Digital Market Index (SensorTower 2024). We identified each resulting advertiser's best-selling product and created ads that linked to pages promoting or allowing users to purchase that product. To promote our ads without violating Meta's terms of use, we created a "product picks" Facebook page for each product category.

Our product categories were shopping, consumer packaged goods, media and entertainment, health and wellness, and food and dining services.[11] The Facebook pages used to promote products in each category were called, respectively, "The Shopping Spot," "Everyday Care Essentials," "Media Roundup," "Health and Wellness Essentials," and "Culinary Crave." These pages are public and viewable on Facebook.[12] Ad creatives used public-source advertising materials to approximate campaigns consumers would likely see. Where relevant, ads link to a site to purchase the advertised product, and otherwise link to a site describing the product in more detail.

Table A1 summarizes the companies, products, and creatives used within each category.

In the remainder of this section, we describe results from a pilot experiment run in January 2025 using Tide creatives.

B.1.2 Experimental design details and estimates.

We first recruited 10 audiences to target in campaigns. To recruit audiences, we ran campaigns targeting US adults aged 18-65. To delineate audiences for retargeting, we used Meta's "Custom Audiences" feature. This feature allows advertisers to identify a set of users based on behaviors or characteristics, such as whether they have previously interacted with another ad or visited an advertisers' website. We used a feature that builds an audience based on users who view at least 25% of a video ad. We made video ads using the 3-second GIFs of the creatives displayed in Table A1, such that users for whom the ad is displayed for 0.75 seconds became part of an audience. We also used the custom audience feature to ensure that the four audiences recruited for each ad were non-overlapping by excluding users from being targeted once they became part of any of the other audiences. The campaigns used to recruit custom audiences were run over four days in January 2025, with a daily budget of $2.

[11] The category with the fifth-highest spending is financial services. Because Meta restricts financial services advertisements, we replace it with food and dining services, the sixth-highest category.

[12] Facebook page IDs are: 61565619873336 (Shopping Spot), 61565078579057 (Everyday Care Essentials), 61564728493259 (Media Roundup), 61565433650990 (Health and Wellness Essentials), and 61565407492274 (Culinary Crave).

We then targeted these audiences with follow-up campaigns that ran for one week and started one week after the initial recruitment campaign began. Across audiences, we experimentally varied ad intensity, ad frequency, and campaign duplication. We designated 5 audiences to target with non-duplicated campaigns, and the remaining 5 to target with duplicated campaigns. Audiences assigned to the non-duplicated condition were targeted by one follow-up campaign. Audiences assigned to the duplicated condition were targeted by two identical follow-up campaigns to induce duplication loss.

The 5 audiences within the non-duplicated and duplicated conditions were assigned to one of five treatments consisting of a daily campaign budget and campaign objective. These conditions were: (i) "low spend, clicks objective", (ii) "mid spend, clicks objective", (iii) "mid spend, reach objective", (iv) "high spend, clicks objective", and (v) "high spend, reach objective." Low, mid, and high spend campaigns received a daily budget of \$2, \$8, and \$12 per day. Campaigns with a clicks objective were given the Meta performance goal of maximizing the number of link clicks, wheres campaigns with a reach objective were given the Meta performance goal of maximizing daily unique reach.

For each campaign, we measured the click-through rate, number of impressions, and campaign reach. We also gathered Meta's estimates of the combined unique reach of campaigns assigned to the duplicated condition, which, along with data on unique reach of each individual campaign, allows us to back out the fraction of a campaign audience that is impressed by both duplicated campaigns. We estimate campaign frequency as the ratio of number of impressions and reach, and campaign intensity as the fraction of the audience impressed by the follow-up campaign.

Using these data, we estimate coefficients in the specification, where a indexes one of the five treatments for audiences described above:

$$RatioCTR_a = \beta_0 + \beta_1 OverlapRatio_a + \beta_2 FreqRatio_a + \beta_3 IntensityRatio_a + \epsilon_a \tag{55}$$

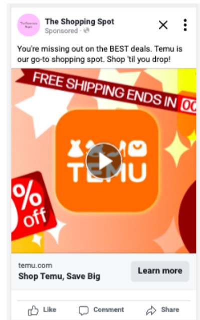
where $RatioCTR_a$ is the ratio of the impression-weighted average click-through rate in duplicated campaigns and the click-through rate in non-duplicated campaigns; $OverlapRatio_a$ is the impression-weighted average fraction of the audience impressed by both campaigns in the duplicated condition; $FreqRatio_a$ is the ratio of the impression-weighted average frequency in the duplicated campaigns to ad frequency in non-duplicated campaigns; and $IntensityRatio_a$ is the ratio of the impression-weighted average intensity in the duplicated campaigns to ad intensity in the non-duplicated campaigns. After estimating coefficients, we estimate $\widehat{RatioCTR}_c$ by setting $OverlapRatio_a = 1$ and the other regressors at their impression-weighted average values. We then form $\hat{L} \equiv \widehat{RatioCTR}_a - \frac{1}{2}$.

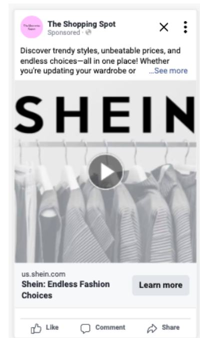
Table A1: Duplication Loss Experiment Creatives

(a) Shopping ads

Company	Amazon	Temu	Shein
Product	Fire Stick	Shopping	Shopping

Ad creative

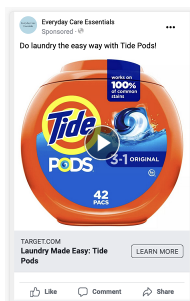


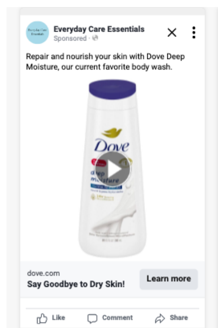


(b) Consumer packaged goods ads

Company	Proctor & Gamble	Unilever	Nestle
Product	Tide Pods	Dove	Nescafe

Ad creative

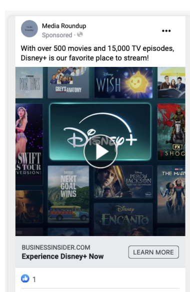




(c) Media and entertainment ads

Company	Disney	NBC Universal	Amazon
Product	Disney+	Peacock	Amazon Prime

Ad creative



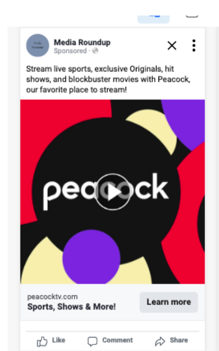
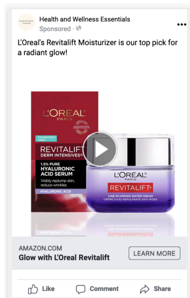
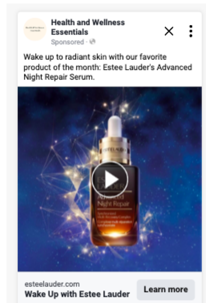
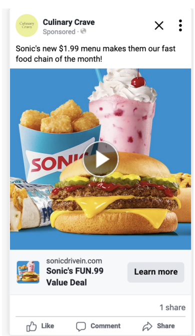
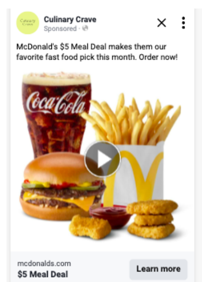
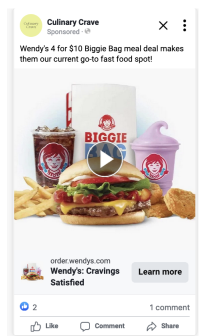


Table A1: Duplication Loss Experiment Creatives, cont.

(d) Health and wellness ads

Company	L'Oreal	Estee Lauder	Olay
Product	Revitalift moisturizer	Night repair serum	Regenerist moisturizer
Ad creative			

(e) Food and dining services ads

Company	Sonic	McDonald's	Wendy's
Product	Sonic's $1.99 menu	$5 meal deal	Biggie Bag
Ad creative			

Notes: This figure describes top companies and products used to develop ad creatives for the duplication experiment. Screen captures of ad creatives are for the video ads used to initially recruit custom audiences, as described in Section B.1.2, but are identical to the static ads used for follow-up campaigns.

B.2 Standard Errors

The covariance matrix of $\mathbf{\Theta}$is

$$\mathbf{\Sigma} = \mathbf{H}^{-1} \cdot \mathbf{\Omega}_h \cdot \mathbf{H}^{-1} \tag{56}$$

where H is:

$$\mathbf{H} = \boldsymbol{R}'_\Theta W \boldsymbol{R}_\Theta \tag{57}$$

Because $\sqrt{n}(\hat{\boldsymbol{\delta}} - \boldsymbol{\delta}) \to_d N(0, \mathbf{\Omega}_\delta)$, according to delta method, $\mathbf{\Omega}_{\boldsymbol{h}}$ is

$$\sqrt{n}\boldsymbol{h}(\Theta_D,\pi) \to_d N(0,\boldsymbol{\Omega_h}) = N(0,\boldsymbol{R}'_\Theta W \boldsymbol{R}_\delta \boldsymbol{\Omega}_\delta \boldsymbol{R}'_\delta W \boldsymbol{R}_\Theta) \tag{58}$$

The matrices $\boldsymbol{R}_\Theta$ and $\boldsymbol{R}_\delta$ represent the Jacobian matrices with respect to the parameters $\boldsymbol{\Theta}$ to be estimated and the empirical moments $\boldsymbol{\delta}$:

$$\boldsymbol{R}_\Theta = \frac{\partial}{\partial \Theta_D}\boldsymbol{h}(\boldsymbol{\Theta},\boldsymbol{\delta}) \tag{59}$$

$$\boldsymbol{R}_\delta = \frac{\partial}{\partial \delta}\boldsymbol{h}(\boldsymbol{\Theta},\boldsymbol{\delta}) \tag{60}$$

A consistent estimator of Σ is:

$$\hat{\boldsymbol{\Sigma}} = \hat{\mathbf{H}}^{-1} \cdot \hat{\boldsymbol{\Omega}}_h \cdot \hat{\mathbf{H}}^{-1} \tag{61}$$

where $\hat{\boldsymbol{\Omega}}_h$ is:

$$\hat{\boldsymbol{\Omega}}_h = \hat{\boldsymbol{R}}'_\Theta \hat{W} \hat{\boldsymbol{R}}_\delta \hat{\boldsymbol{\Omega}}_\delta \hat{\boldsymbol{R}}'{}_\delta \hat{W} \hat{\boldsymbol{R}}_\Theta \tag{62}$$

and $\hat{H}$ is:

$$\hat{\mathbf{H}} = \hat{\boldsymbol{R}}'_\Theta \hat{W} \hat{\boldsymbol{R}}_\Theta \tag{63}$$

and $\hat{\boldsymbol{\Omega}}_\delta$ is:

$$\hat{\boldsymbol{\Omega}}_\delta = \begin{pmatrix} \hat{\boldsymbol{\Omega}}_B & & & & \\ & \hat{\boldsymbol{\Omega}}_D & & & \\ & & \hat{\boldsymbol{\Omega}}_G & & \\ & & & \hat{\boldsymbol{\Omega}}_P & \\ & & & & \hat{\boldsymbol{\Omega}}_E \end{pmatrix}$$

where $\hat{\boldsymbol{\Omega}}_B$ is the sample covariance matrix for $\left(\left\{\hat{T}^C_{kj}\right\}_{k,j}, \left\{\hat{T}^C_j, \hat{T}^B_j\right\}_j\right)$; $\hat{\boldsymbol{\Omega}}_D$ is the sample covariance matrix for $\left(\left\{\hat{T}^C_{mj}\right\}_j, \left\{\hat{T}^{Dj'}_{mj}\right\}_j\right)$, $\hat{\boldsymbol{\Omega}}_G$ is the variance of $\widehat{\frac{\partial T}{\partial \alpha}\frac{\alpha}{T}}$, $\hat{\boldsymbol{\Omega}}_P$ is the sample covariance matrix for $\left(\hat{P}_F\right)$, and $\hat{\boldsymbol{\Omega}}_E$ is the sample covariance matrix for $\left(\left\{\hat{\mathcal{E}}^2_{kj}\right\}_{j,k}, \widehat{\mathcal{E}}_{mFI}\right)$. This assumes no covariance in moments across experiments – for example, between DA, FIES, and Goli et al. (2018).

Since our system is just-identified, we use $\hat{W} = I$. We compute the Jacobians of $\boldsymbol{h}(\cdot)$ using MATLAB symbolic differentiation.

C Counterfactuals Appendix

C.1 Welfare formulas

Let O be the number of consumers with positive time use on both platforms. Consumer surplus, as a function of $\boldsymbol{T}_i$, is:

$$\sum_i U_i(\boldsymbol{T}_i) = O \cdot \left[\sum_j (\xi_{mj} - \gamma\alpha_j) T_{mj} + E[\varepsilon_{ij} e_{ij} | k = m] - \sigma_j/2 \left(T_{mj}^2 + \mathcal{E}_{mj}^2\right) + \rho \left(T_{m1}T_{m2} + \mathcal{E}_{m12}\right)\right]$$
$$+ \sum_j (N_j - O) \cdot \left[(\xi_{sj} - \gamma\alpha_j) \cdot T_{sj} + E[\varepsilon_{ij} e_{ij} | k = s] - \sigma_j/2 \left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)\right]$$

Proudcer surplus in the merged equilibrium is:

$$\begin{aligned} R^m(\boldsymbol{\alpha}) &= \eta \cdot \sum_i \boldsymbol{\alpha} \cdot \boldsymbol{T}_i \cdot \left(1 + \eta_0 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T}_i}{Am}\right) \\ &= \eta \cdot O \cdot \left[\boldsymbol{\alpha} \cdot \boldsymbol{T}_m \cdot (1 + \eta_0) - \frac{(\boldsymbol{\alpha} \cdot \boldsymbol{T}_m)^2 + \left(\alpha_1^2 \mathcal{E}_{s1}^2 + \alpha_2^2 \mathcal{E}_{s2}^2 + 2 \cdot \alpha_1 \alpha_2 \mathcal{E}_{m12}\right)}{Am}\right] \\ &+ \eta \cdot \sum_j (N_j - O) \cdot \left[\alpha_j T_{sj} \cdot (1 + \eta_0) - \frac{\alpha_j^2 \left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)}{Am}\right] \end{aligned} \tag{64}$$

Producer surplus in the separated equilibrium is:

$$\begin{aligned} R_j^s(\boldsymbol{\alpha}) &= \eta \cdot \left(1 - \zeta_j O'_{aj}(\boldsymbol{\alpha})\right) \cdot \sum_i \alpha_j T_{ij} \cdot \left(1 + \eta_0 - \frac{\alpha_j T_{ij}}{Am}\right) \\ &= \eta \cdot \left(1 - \zeta_j O'_{aj}(\boldsymbol{\alpha})\right) \cdot O \cdot \left[\alpha_j T_{mj}(1 + \eta_0) - \frac{\alpha_j^2 \left(T_{mj}^2 + \mathcal{E}_{mj}^2\right)}{Am}\right] \\ &+ \eta \cdot \left(1 - \zeta_j O'_{aj}(\boldsymbol{\alpha})\right) \cdot (N_j - O) \cdot \left[\alpha_j T_{sj}(1 + \eta_0) - \frac{\alpha_j^2 \left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)}{Am}\right] \end{aligned}$$

Advertiser surplus in the merged equilibrium is:

$$\begin{aligned} AS^m(\boldsymbol{\alpha}^m) &= \sum_i Am \cdot \int_{p_i(\boldsymbol{\alpha}^m)}^{(\pi\omega)^{\max}} x dH(x) \\ &= \sum_i \left[\frac{A}{2\eta} \eta^2 (1 + \eta_0)^2 - \frac{A}{2\eta} p_i^2 - \boldsymbol{\alpha} \cdot \boldsymbol{T}_i \cdot p_i\right] \end{aligned}$$

We implement this by calculating:

$$\sum_i \frac{A}{2\eta}\eta^2\left(1+\eta_0\right)^2 = N\frac{A}{2\eta}\eta^2\left(1+\eta_0\right)^2;$$

$$\begin{aligned}\sum_i p_i^2 &= \sum_i \eta^2 \cdot \left(1+\eta_0 - \frac{\boldsymbol{\alpha}\cdot\boldsymbol{T}_i}{Am}\right)^2 \\ &= \eta^2 \cdot O \cdot \left[(1+\eta_0)^2 - 2\cdot\frac{1+\eta_0}{Am}\cdot\boldsymbol{\alpha}\cdot\boldsymbol{T}_m + \frac{(\boldsymbol{\alpha}\cdot\boldsymbol{T}_m)^2 + \left(\alpha_1^2\mathcal{E}_{s1}^2 + \alpha_2^2\mathcal{E}_{s2}^2 + \alpha_1\alpha_2\mathcal{E}_{m12}\right)}{(Am)^2}\right] \\ &+ \eta^2 \cdot \sum_j (N_j - O)\cdot\left[(1+\eta_0)^2 - 2\cdot\frac{1+\eta_0}{Am}\cdot\alpha_j T_{sj} + \frac{\alpha_j^2\left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)}{(Am)^2}\right];\end{aligned}$$

$$\sum_i \boldsymbol{\alpha}\cdot\boldsymbol{T}_i\cdot p_i = R^m(\boldsymbol{\alpha}).$$

where $R^m(\boldsymbol{\alpha})$ is given by Equation (64).

To find advertiser surplus in the separated equilibrium, first calculate advertiser surplus with no overlap, then subtract out lost surplus from overlap. Advertiser surplus without overlap is:

$$\begin{aligned}AS_j'(\boldsymbol{\alpha}) &= \sum_{i\in\mathcal{U}_j} Am\cdot\int_{p_{ij}(\boldsymbol{\alpha})}^{(\pi\omega)^{\max}} x\,dH(x) \\ &= \sum_{i\in\mathcal{U}_j}\left[\frac{A}{2\eta}\eta^2(\eta_0+1)^2 - \frac{A}{2\eta}\left(\frac{p_{ij}}{1-\zeta_j O_{aj}'}\right)^2 - \alpha_j T_{ij} p_{ij}\right]\end{aligned}$$

We implement this by calculating:

$$\sum_{i\in\mathcal{U}_j}\frac{A}{2h}\eta^2(\eta_0+1)^2 = N_j\cdot\frac{A}{2h}\eta^2(\eta_0+1)^2$$

$$\begin{aligned}
\sum_{i\in\mathcal{U}_j}\frac{p_{ij}^2}{\left(1-\zeta_j O'_{aj}\right)^2} &= \sum_{i\in\mathcal{U}_j}\eta^2\cdot\left(1+\eta_0-\frac{\alpha_j T_{ij}}{Am}\right)^2 \\
&= \eta^2\cdot O\cdot\left((1+\eta_0)^2 - 2\cdot\frac{1+\eta_0}{Am}\alpha_j T_{mj} + \frac{\alpha_j^2\cdot\left(T_{mj}^2+\mathcal{E}_{mj}^2\right)}{(Am)^2}\right) \\
&+ \eta^2\cdot(N_j-O)\cdot\left((1+\eta_0)^2 - 2\cdot\frac{1+\eta_0}{Am}\alpha_j T_{sj} + \frac{\alpha_j^2\cdot\left(T_{sj}^2+\mathcal{E}_{sj}^2\right)}{(Am)^2}\right)
\end{aligned}$$

$$\begin{aligned}
\sum_{i\in\mathcal{U}_j}\alpha_j T_{ij}p_{ij} &= \eta\cdot\left(1-\zeta_j O'_{aj}\right)\cdot\sum_{i\in\mathcal{U}_j}\alpha_j T_{ij}\cdot\left(1+\eta_0-\frac{\alpha_j T_{ij}}{Am}\right) \\
&= \eta\cdot\left(1-\zeta_j O'_{aj}\right)\cdot O\cdot\left[\alpha_j T_{mj}(1+\eta_0) - \frac{\alpha_j^2\left(T_{mj}^2+\mathcal{E}_{mj}^2\right)}{Am}\right] \\
&+ \eta\cdot\left(1-\zeta_j O'_{aj}\right)\cdot(N_j-O)\cdot\left[\alpha_j T_{sj}(1+\eta_0) - \frac{\alpha_j^2\left(T_{sj}^2+\mathcal{E}_{sj}^2\right)}{Am}\right]
\end{aligned}$$

To compute advertiser surplus given overlap, first note that an advertiser buys enough clicks on j so that they will serve an ad to all i where:

$$p_{ij}\le\pi_a\omega_{ia}\left(1-\zeta_j\frac{\partial O_a}{\partial q_j}\right)\iff H_i^{-1}\left(1-\frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{A}\right)\le\pi_a\omega_{ia}$$

Overlap occurs if the advertiser would buy enough clicks to serve i on both j and $-j$. Define $\underline{p}_i(\boldsymbol{\alpha}) = \max_j H_i^{-1}\left(1-\frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{A}\right)$. The surplus lost due to overlap for person i is

$$\begin{aligned}
\sum_i\int_{\underline{p}_i(\boldsymbol{\alpha})}^{(\pi\omega)^{\max}} x\,dH(x) &= Am\cdot E\left[\pi_a(\omega_{ia}-\underline{\omega}_{ia})|\pi_a\omega_{ia}\ge\underline{p}_i(\boldsymbol{\alpha})\right] \\
&= Am\cdot E\left[(1-\kappa\cdot\rho_i)\cdot\pi_a\omega_{ia}|\pi_a\omega_{ia}\ge\underline{p}_i(\boldsymbol{\alpha})\right] \\
&= Am\cdot\left(1-\kappa\cdot E\left[\rho_i|\pi_a\omega_{ia}\ge\underline{p}_i(\boldsymbol{\alpha})\right]\right)\cdot E\left[\pi_a\omega_{ia}|\pi_a\omega_{ia}\ge\underline{p}_i(\boldsymbol{\alpha})\right] \\
&= \frac{A}{2\eta}\cdot(1-\kappa\cdot E[\rho_i])\cdot\left[\eta^2(1+\eta_0)^2-\left(\underline{p}_i(\boldsymbol{\alpha})\right)^2\right]
\end{aligned}$$

In this derivation, the third line follows from the second because ρ_i is a linear transformation of $p_i^m = H_i^{-1}\left(1-\frac{\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})}{Am}\right)$, which is uncorrelated with $\pi_a\omega_{ia}$ by Assumptions 1 and 2. This also implies that conditioning on $\pi_a\omega_{ia}\ge\underline{p}_i(\boldsymbol{\alpha})$ does not change the expectation of ρ_i, in which case hte fourth line follows from the third.

Total overlap becomes:

$$AS^o(\boldsymbol{\alpha}) = (1 - \kappa \cdot E[\rho_i]) \cdot \sum_{i \in \mathcal{U}_m} \frac{A}{2\eta} \left[\eta^2 (1+\eta_0)^2 - \left(\max_j \eta \cdot \left(1 + \eta_0 - \frac{\alpha_j T_{ij}(\boldsymbol{\alpha})}{A} \right) \right)^2 \right] \tag{65}$$

where:

$$E[\rho_i] = \frac{1}{(1+\eta_0)} \cdot \left(1 + \eta_0 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T}_m(\boldsymbol{\alpha})}{Am} \right)$$

Given heterogeneous time use among multi-homers, it is challenging to get an analytical expression for the above. Instead, we directly integrate by following these steps:

1. Write a function mapping from parameters of time use heterogeneity to $\underline{p}_i(\boldsymbol{\alpha}; \boldsymbol{\Xi})$.
2. Numercially integrate under this function using our grid describing the distribution of $\boldsymbol{\Xi}$.
3. Use Equation (65) to calculate $AS^o(\boldsymbol{\alpha})$.

Finally, we calculate $AS^s_j(\boldsymbol{\alpha}) = \sum_j AS'_j(\boldsymbol{\alpha}) - AS^o(\boldsymbol{\alpha})$.

C.2 Miscellaneous formulas

C.2.1 Average price per impression

Average price per impression on platform j is:

$$\overline{p}_j = \frac{\sum_{i \in \mathcal{U}_j} p_{ij} \alpha_j T_{ij}}{\sum_{i \in \mathcal{U}_j} \alpha_j T_{ij}}$$

The denominator is straightforward to calculate. In the merged equilibrium, the numerator is:

$$\begin{aligned} \sum_{i \in \mathcal{U}_j} p_i \alpha_j T_{ij} &= \sum_{i \in \mathcal{U}_j} \eta \cdot \left(1 + \eta_0 - \frac{\boldsymbol{\alpha} \cdot \boldsymbol{T}_i}{Am} \right) \alpha_j T_{ij} \\ &= O \cdot \eta \cdot \left[(1+\eta_0)\, \alpha_j T_{mj} - \frac{\alpha_j^2 \left(T_{mj}^2 + \mathcal{E}_{mj}^2 \right) + \alpha_j \alpha_{-j} \left(T_{mj} T_{m,-j} + \mathcal{E}_{12} \right)}{Am} \right] \\ &+ (N_j - O) \cdot \eta \cdot \left[(1+\eta_0)\, \alpha_j T_{sj} - \frac{\alpha_j^2 \left(T_{sj}^2 + \mathcal{E}_{sj}^2 \right)}{Am} \right] \end{aligned} \tag{66}$$

In the separated equilibrium, the numerator is:

$$\begin{aligned}
\sum_{i\in\mathcal{U}_j} p_{ij}\alpha_j T_{ij} &= (1-\zeta O'_{aj})\cdot\eta\cdot\sum_{i\in\mathcal{U}_j}\left(1+\eta_0-\frac{\alpha_j T_{ij}}{Am}\right)\alpha_j T_{ij} \\
&= (1-\zeta O'_{aj})\cdot\eta\cdot O\cdot\left((1+\eta_0)\,\alpha_j T_{mj}-\frac{\alpha_j^2\left(T_{mj}^2+\mathcal{E}_{mj}^2\right)}{Am}\right) \\
&+ (1-\zeta O'_{aj})\cdot\eta\cdot(N_j-O)\cdot\left((1+\eta_0)\,\alpha_j T_{sj}-\frac{\alpha_j^2\left(T_{sj}^2+\mathcal{E}_{sj}^2\right)}{Am}\right)
\end{aligned}$$

C.2.2 Formula for $\zeta_j(\boldsymbol{\alpha})$

By definition:

$$\begin{aligned}
\zeta_j(\boldsymbol{\alpha}) &= 1-\kappa\cdot p_i(\boldsymbol{\alpha}^m)/(\eta\cdot(1+\eta_0) \\
&= 1-\frac{\kappa}{\eta\cdot(1+\eta_0)}\frac{E_i\left[T_{ij}(\boldsymbol{\alpha})\cdot p_i^m|i\in\mathcal{U}_m\right]}{T_{mj}}
\end{aligned}$$

Since:

$$\begin{aligned}
O^{-1}\sum_{i\in\mathcal{U}_m} T_{ij}(\boldsymbol{\alpha})\cdot p_i^m &= O^{-1}\sum_{i\in\mathcal{U}_m}\left[\eta\cdot\left(1+\eta_0-\frac{\boldsymbol{\alpha}^m\cdot\boldsymbol{T}_i(\boldsymbol{\alpha}^m)}{Am}\right)\cdot T_{ij}(\boldsymbol{\alpha})\right] \\
&= \eta\cdot\left[(1+\eta_0)\cdot T_{mj}(\boldsymbol{\alpha})-\frac{\alpha_j^m\left(T_{mj}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{mj}^2\right)+\alpha_{-j}^m\left(T_{m,-j}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{12}\right)}{Am}\right]
\end{aligned}$$

the formula for $\zeta_j(\boldsymbol{\alpha})$ is

$$\zeta_j(\boldsymbol{\alpha}) = 1-\frac{\kappa}{1+\eta_0}\cdot T_{mj}^{-1}\left((1+\eta_0)\cdot T_{mj}(\boldsymbol{\alpha})-\frac{\alpha_j^m\left(T_{mj}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{mj}^2\right)+\alpha_{-j}^m\left(T_{m,-j}(\boldsymbol{\alpha}^m)T_{mj}(\boldsymbol{\alpha})+\mathcal{E}_{12}\right)}{Am}\right) \tag{67}$$

C.2.3 Aggregate elasticity of ad demand in merged equilibrium

Rewrite the merged platform problem as:

$$\max_{\boldsymbol{\alpha}}\sum_i p_i(\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha}))\cdot\boldsymbol{\alpha}\cdot\boldsymbol{T}_i(\boldsymbol{\alpha})$$

In this notation, the FOC with respect to α_j is

$$0 = \sum_i \frac{\partial}{\partial \alpha_j} \left(\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})\right) \cdot \left(p_i + \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\partial p_i}{\partial \left(\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})\right)}\right)$$
$$= \sum_i \left(T_{ij}(\boldsymbol{\alpha}) + \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}_i}{\partial \alpha_j}\right) \cdot \left(p_i - \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\eta}{Am}\right)$$

Rearranging:

$$-\sum_i \boldsymbol{\alpha} \cdot \frac{\partial \boldsymbol{T}_i}{\partial \alpha_j} = \sum_i T_{ij}(\boldsymbol{\alpha}) \cdot \left(p_i - \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot \frac{\eta}{Am}\right) \tag{68}$$

This illustrates the typical two-sided market intuition that the merged platform balances the elasticity of time use (LHS) with the elasticity of demand from advertisers (RHS). Define the *aggregate elasticity of ad demand* on platform j as:

$$\varepsilon_j^D(\boldsymbol{\alpha}) \equiv -\frac{Am}{\eta} \frac{\sum_i T_{ij}(\boldsymbol{\alpha}) \cdot p_i}{\sum_i T_{ij}(\boldsymbol{\alpha}) \cdot \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha})} \tag{69}$$

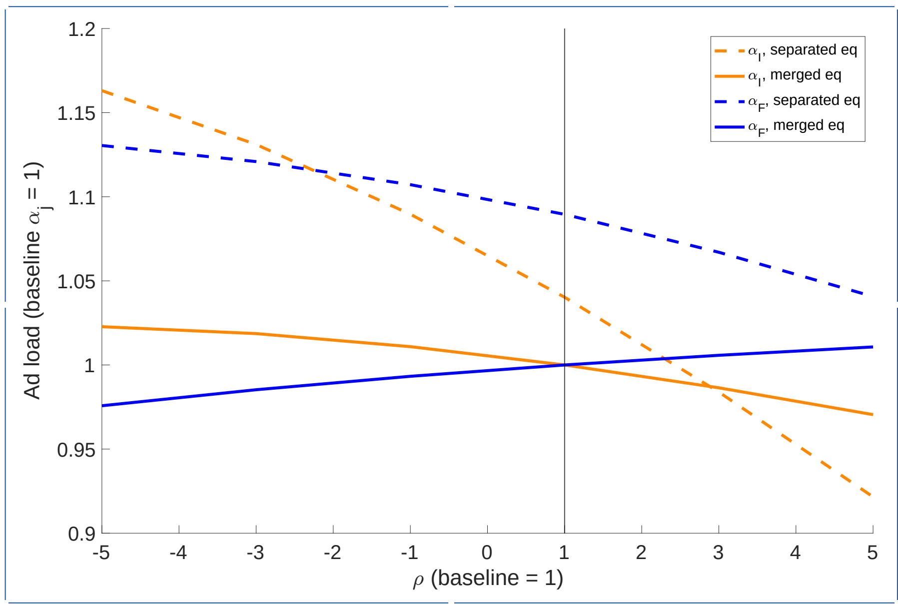
where we refer to $\varepsilon^D(\boldsymbol{\alpha})$ as an elasticity because when users are homogeneous and the market is one-sided (users don't care about ads), the expression reduces to $\varepsilon^D(\boldsymbol{\alpha}) = \frac{\partial \log(\boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}))}{\partial \log p}$, and quantity choice follows the inverse elasticity rule. The platform cares about the user side of the market as well; RHS of equation (68) is positive, which means the aggregate elasticity of ad demand is above one. The more elastic the user side of the market is, the more above one is the absolute value of the aggregate elasticity of advertiser demand.

To compute equation (69), the numerator is the same as equation (66), divided through by α_j. The denominator is:

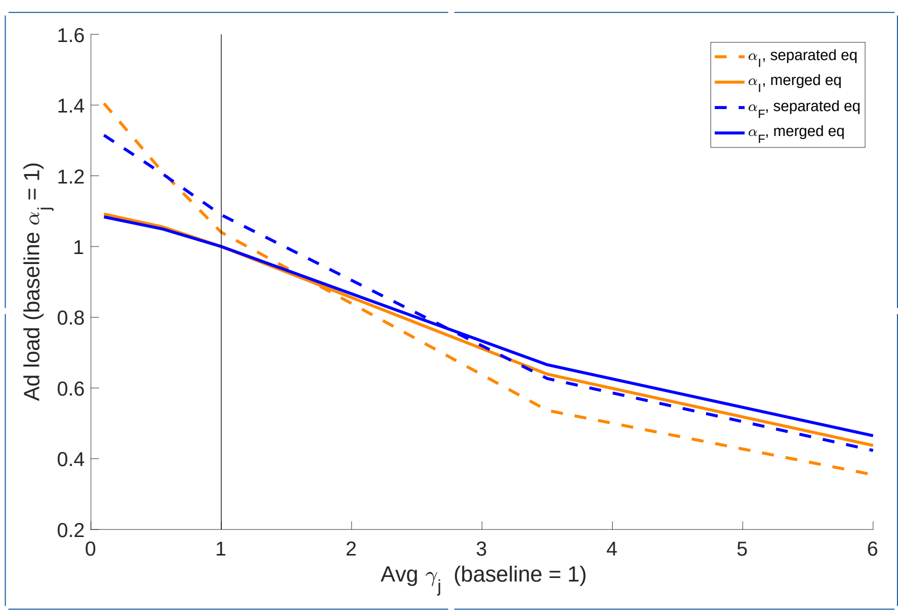
$$\sum_{i \in \mathcal{U}_j} \boldsymbol{\alpha} \cdot \boldsymbol{T}_i(\boldsymbol{\alpha}) \cdot T_{ij}(\boldsymbol{\alpha}) = O \cdot \left[\alpha_j \cdot \left(T_{mj}^2 + \mathcal{E}_{mj}^2\right) + \alpha_{j'} \cdot \left(T_{mj} T_{mj'} + \mathcal{E}_{12}\right)\right]$$
$$+ (N_j - O) \cdot \left[\alpha_j \cdot \left(T_{sj}^2 + \mathcal{E}_{sj}^2\right)\right]$$

C.3 Additional Counterfactual Figures

Figure A1: **Counterfactual Ad Load Sensitivity: User-Side Parameters**



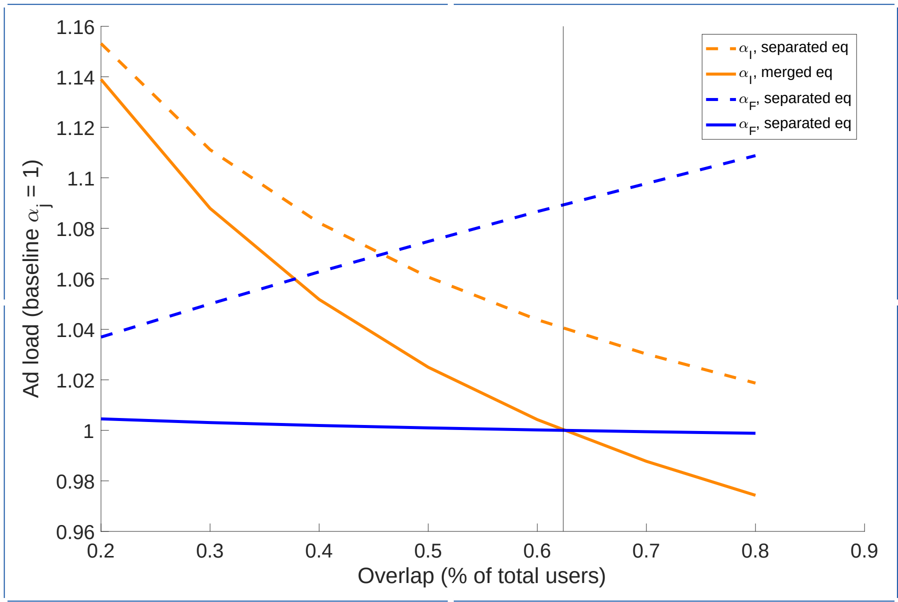
(a) **Platform Substitution**



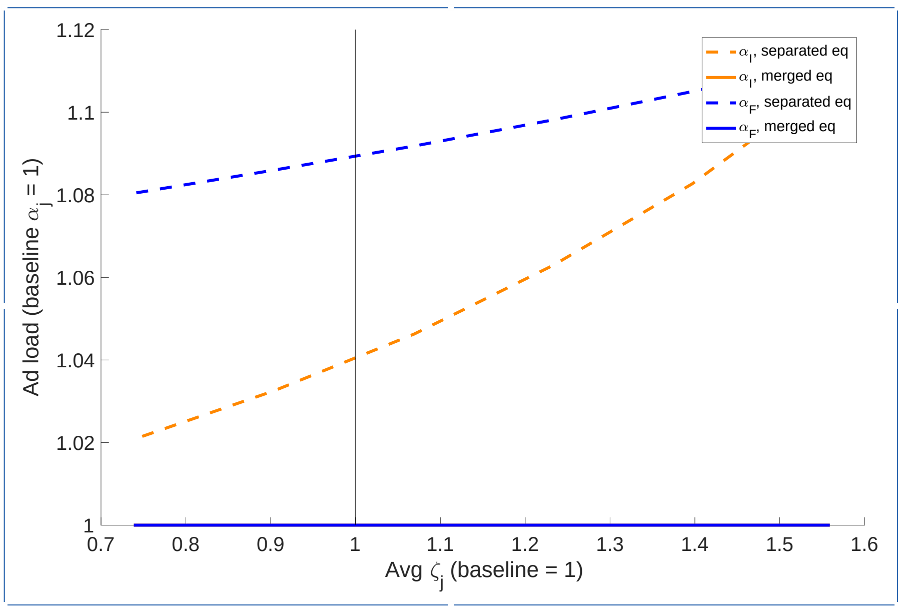
(b) **Ad Disutility**

Notes: This figure presents the impact of alternative user-side parameters on estimates of the total surplus effects of a Facebook-Instagram separation. We perturb the parameter on the horizontal axis, compute ad load in the new merged and separated equilibria, and and plot ad load on each platform relative to its value in the merged equilibrium under baseline parameter estimates. Higher ρ indicates that platforms are stronger substitutes, and higher average γ_j implies that users are more averse to ad load.

Figure A2: **Counterfactual Total Surplus Sensitivity: User-Side Parameters**



(a) **Overlap**



(b) **Loss from Duplicated Impressions**

Notes: This figure presents the impact of alternative advertiser-side parameters on estimates of the total surplus effects of a Facebook-Instagram separation. We perturb the parameter on the horizontal axis, compute ad load in the new merged and separated equilibria, and plot ad load on each platform relative to its value in the merged equilibrium under baseline parameter estimates. Higher overlap maintains the ratio of single-homers on Facebook to Instagram, and varies the total number of multi-homers as a fraction of total users. Higher average ζ_j indicates a greater loss from duplication, with the range of plotted ζ_j generated from $\kappa \in [0, 1]$.