Platform Competition and App Development*

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Abstract

We study the development of apps on competing platforms. We show that competition leads to commissions exceeding those maximizing consumer surplus (and, a fortiori, social welfare) whenever raising one commission reduces rivals' app bases. We relate this finding to economies of scope in app development and, to illustrate it, consider a setting in which some developers can port their apps at no cost: as their proportion increases, app development is progressively choked-off.

Fostering platform competition or interoperability may therefore fail to produce the desired results. Within-platform app store competition, together with appropriate access conditions, may constitute a more promising avenue.

Keywords: Platform competition, ad-valorem commissions, app stores, app development.

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1 Introduction

The 30 percent commission charged by Apple and Google has prompted major disputes, such as the legal battle led by Epic Games,[1] and triggered policy initiatives around the world. For instance, to reduce the commissions paid by app developers, the South Korean parliament adopted in 2021 a bill banning major app store operators – such as Google and Apple – from requiring developers to only use the app stores' payment systems. Later on, the Indian Competition Commission issued a similar order.[2] In 2022, the European Union adopted the Digital Markets Act, which requires gatekeepers to apply fair, reasonable and non-discriminatory conditions of access to app stores, among others.[3]

The main concern about Apple's and Google's commissions is their negative impact on app development; a crisp summary was provided by Brent Simmons, a Mac and iOS app developer, in his testimony before the U.S. Congress:

"[T]he more money Apple takes from developers, the fewer resources developers have. .... They decide not to make apps at all that they might have made were it easier to be profitable."[4]

In response, Apple and Google argue that platforms and consumers have a common interest in attracting apps, and moreover point to the disciplining role of platform competition. For instance, in its response to the investigation of the Dutch National Competition Authority (NCA), Google argues:

"The level of the commission fee charged is used by app stores to compete with each other, as a means to attract app providers on their platform."[5]

Regulators have however expressed doubt about the extent to which the largest platforms are subject to competitive pressure and pointed instead to substantial switching costs and behavioral biases among consumers – see, e.g., U.K. Competition and Markets Authority (2022) and U.S. Department of Justice (2024).

To shed some light on this debate, we study a setting in which two-sided platforms compete on prices for consumers and on ad valorem commissions for apps. Consumers single-home and benefit from the platform's service and the available apps, whereas app developers, who derive their revenue from consumers but face heterogeneous in-

[1] In August 2020, Epic started encouraging mobile-app users of its Fortnite game to adopt Epic's payment option, offering a 20% discount from Apple's or Google's in-app purchase. In response, Apple and Google removed Fortnite from their respective app stores, which led Epic to sue Apple and Google, with the backing of Microsoft, Facebook, Spotify, Match Group and ten other companies.

[2] See https://cci.gov.in/images/pressrelease/en/pr-no-562022-231666698260.pdf.

[3] See Article 6.12.

[4] See Subcommittee (2020), at p. 350. In the same vein, see Greg Bensinger's article, "What Apple's Fortnite Fee Battle Is Really About," the New York Times, https://www.nytimes.com/2020/09/24/opinion/apple-google-mobile-apps.html.

[5] Netherlands Authority for Consumers and Markets (2019), at p. 92.

vestment costs, may single- or multihome.

Our main finding is that platform competition may not be a cure but, rather, an *obstacle* to app development. Specifically, we find that, where the commissions maximizing consumer surplus would also maximize platforms' joint profit (but exceed those favored by developers), competition generates instead higher commissions whenever an increase in one commission reduces the number of apps present on the *rival* platform. This, in turn, occurs whenever there are (supply-side or demand-side) economies of scope in app development, as investment decisions are then more likely to be driven by both platforms' commissions.

Our analysis thus highlights a key factor, namely, the extent to which app development is driven by the overall business opportunities offered by the two platforms. In practice, due to the lack of consumer multihoming on the consumer side, successful apps tend indeed to be present in both platforms; for instance, the U.K. Competition and Markets Authority notes:

"Most large and popular third-party apps are present on both Apple's iOS and Google's Android. For example, we have estimated that 85% of the top 5,000 apps on the App Store also list on the Play Store and vice versa."[6]

In a similar vein, the Dutch NCA finds that consumers' initial choice between an iPhone and an Android phone does not depend on the availability of apps, because all popular and known apps are present in both smartphone platforms.[7] Furthermore, the recent complaint of the U.S. Department of Justice (2024) against Apple points to "intrinsically multihoming" apps which, because of restrictions imposed by Apple, were not developed either for Android phones.[8]

The Chinese app store market provides further evidence on the relation between platform competition and the commissions charged to developers. In that market, where Google Play Store is not available and Apple has less than 20 percent market share, there is vivid competition among multiple Chinese smartphone manufacturers, all based on the Android operating system. Yet, the major Chinese manufacturers (e.g., Xiaomi, Oppo, Vivo, and Huawei), who own their own app stores, charge a 50 percent commission to app developers.[9] Hence, intense platform competition is associated with even higher commissions than those charged by Google and Apple. This is in line with our analysis, as we find that increasing the number of platforms amplifies the gap between the equilibrium commission and the consumer-surplus maximizing one and

[6]U.K. CMA (2022), p. 121.

[7]See Netherlands Authority for Consumers and Markets (2019).

[8]See the discussion at the end of Section 3.

[9]See for instance "China's App Store Fee's Make Apple's Look Cheap" by Zheping Huang, Bloomberg, 8 October, 2020, https://www.bloomberg.com/news/newsletters/2020-10-08/china-s-app-store-fees-make-apple-s-look-cheap.

further discourages the development of multihoming apps – by contrast, the degree of substitution among platforms appears to play a less important role.

These insights suggest that platform competition may not discipline the commissions charged to developers.[10] In the same vein, policies designed to foster platform interoperability – and, thus, economies of scope in app development – may result in higher commissions.

Because Apple and Google have charged the same commission since the launch of their app stores, throughout the paper we assume that platforms compete for apps before competing for consumers. That is, in a first stage, platforms set their commissions, and in response developers make their investment decisions. Therefore, when competing later on for consumers, each platform benefits from a per-consumer *subsidy*, corresponding to the value – for the platform and its consumers – generated by the apps available on the platform. It follows that platforms' joint interest is aligned with consumers' own interests, as larger subsidies result in both greater profits and lower prices; maximizing industry profit or consumer surplus thus boils down to maximizing the subsidies. By contrast, app developers favor lower commissions; hence, maximizing social welfare requires lower commissions than those that maximize consumer surplus or the platforms' profits.

To compare these benchmarks with the competitive outcome, we begin with a stylized approach that imposes minimal assumptions on developers' response to the commissions, and on the prices and profits stemming from the resulting subsidies. If a platform's commission does not influence its rival's app base, it does not affect the rival's subsidy either. In this case, in equilibrium each platform selects the commission that maximizes its own subsidy. Consequently, consumer surplus is also maximized. If instead an increase in one platform's commission decreases (resp., increases) the rival's app base, the platforms have an additional incentive to raise (resp., lower) their commissions, in order to make their rival less aggressive. As a result, competition leads to commissions that are above (resp., below) the level that maximizes their own subsidy and, consequently, consumer surplus.

We relate these insights to the existence of (dis-)economies of scope in app development. When development costs are independent across platforms, raising a platform's commission has no impact on the rival's app base; the competitive outcome therefore maximizes consumer surplus. When instead there are economies (resp., diseconomies) of scope, raising a platform's commission reduces (resp., increases) the rival's app base; the equilibrium commissions then exceed (resp., lie below) the level maximizing consumer surplus.

[10] Within-platform competition between third-party app stores may provide a more promising avenue.

We also note that our insights carry over when platforms charge wholesale prices rather than commissions, but no longer hold when instead they charge fixed fees. This is because wholesale prices, like ad valorem commissions, affect platforms' profit only through their subsidies when competing for consumers. By contrast, fixed fees have a direct impact on platforms' profits, other than through the subsidies. As a result, platforms' joint interest is no longer aligned with consumers'.

To gain further insights, we consider a particular setting with horizontal differentiation à la Hotelling on the consumer side and two types of developers on the app side: *independent decision markers* face platform-specific development costs, whereas *joint decision makers* can port their apps across platforms at no cost, and thus base their development decisions on the overall profitability offered by the two platforms. We show that, as long as there is a positive proportion of joint decision makers, platform competition leads to a higher commission than what would maximize consumer surplus. Furthermore, as this proportion tends to one, the commissions become so high that the app development is progressively choked-off. The intuition is that a platform has little incentive to encourage the development of apps when most of these apps become also available on the competing platform. By contrast, a monopolistic firm running both platforms would seek to encourage app development – and actually choose the commissions that maximize consumer surplus. Interestingly, neither the monopolistic nor the competitive levels of the commissions depend on the degree of substitution between the two platforms.

Finally, we extend our analysis in two directions. In section 5.1, we consider a generalization of the Hotelling setting that accommodates an arbitrary number of competing platforms. The degree of substitution between the platforms has again no impact on the commissions, but increasing the *number* of competing platforms raises further the commissions. In section 5.2, we introduce uncertainty about the popularity of apps and allow them to be ported if they are successful. We show that platform competition leads again to commissions exceeding the level that maximizes consumer surplus whenever popular apps (which eventually become available on both platforms) play a significant role, as is the case in practice.

Related literature. To study developers' innovation incentives, we build on the model of competitive bottlenecks developed by Armstrong (2006), with single-homing on one side of the market, and multihoming with independent participation decisions on the other side. A key finding is that, in equilibrium, the number of users on the latter side maximizes the joint surplus of the platform and its other users. Armstrong and Wright (2007) find conditions under which single-homing on one side and multihoming on the other side arise endogenously. Belleflamme and Peitz (2010) extend Armstrong (2006) by allowing sellers to invest and improve their offerings, and find that sellers invest less than is socially desirable when sellers multihome and buyers single-home. Choi and Jeon (2022) study platform design in a model of competitive bottlenecks and identify the design biases (e.g., in the direction of innovation) generated by different platform business models. Teh and Wright (2023) extend Armstrong's competitive bottleneck model to an oligopoly setting in which platforms can use multiple instruments on the multihoming side. They focus on symmetric equilibria featuring full coverage and show that Armstrong's original insight holds in the absence of spillovers on the rival platforms. Armstrong's competitive bottleneck result has also been revisited by Etro (2023), who studies sequential competition in a setting corresponding to our Hotelling model with independent decision makers, and shows that platform competition then leads to commissions that maximize consumer surplus; the platforms' profits being independent of the level of the commissions in the Hotelling setting, this extends Armstrong's insight to a context of sequential competition.

We contribute to this literature in three ways. First, we further extend Armstrong's insight to more general sequential settings, with only minimal assumptions on app supply and consumer demand (in particular, we allow the total demand to be elastic). Specifically, the literature on competitive bottlenecks usually considers simultaneous pricing decisions on both sides, followed by simultaneous participation decisions on both sides. Our interest on app stores in mobile platforms leads us to focus instead on a setting in which pricing and development decisions on the app side take place before platforms' competition for consumers. As noted above, this competition is then driven by the subsidies generated by the apps available on each platform and, as a result, platforms' and consumers' interests are fully aligned – they all wish to maximize these subsidies. Furthermore, if app development decisions are independent across platforms, then the commission set by a platform only affects the platform's own subsidy, and Armstrong's insight carries over – with the twist that competition leads to commissions that maximize *both* platforms' profit *and* consumer surplus, rather than only the sum of them.

Second, we show that Armstrong's insight no longer holds when development decisions are interdependent: platform competition yields instead higher (resp., lower) commissions when developing an app for one platform encourages (resp., discourages) its development for the other platform. We further related this to the presence of (dis-)economies of scope. Third, we show that these insights carry over when platforms compete in wholesale prices instead of ad valorem commissions, but no longer hold if they compete instead in fixed fees; in particular, competition fails to maximize platforms' and consumers' joint payoff even when development decisions are independent.

An earlier related paper is Wright (2002), who studies the market for fixed-to-mobile calls. Mobile network operators (MNOs) compete to attract consumers and charge fixed-to-mobile termination fees to a fixed-line network operator. If that operator is constrained to charge the same price for all fixed-to-mobile calls, then the MNOs set termination fees higher than the monopoly fee; in particular, if two MNOs compete à la Hotelling, the termination fees are so high that there are no fixed-to-mobile calls, a finding similar to our choke-off result. However, several differences can be noted. First, mobile subscribers are supposed to derive zero utility from fixed-to-mobile calls and are thus insensitive to the level of the termination fees. By contrast, in our setting consumers enjoy the applications and thus indirectly care about the commissions charged on the app side as well as about the device prices on the consumer side. Second, the choke-off of fixed-to-mobile calls stems from a non-discrimination rule imposed on the fixed-line network, whereas in our setting, the choke-off of app development arises instead when all developers make joint development decisions.[11]

Anderson and Bedre-Defolie (2024) consider a monopoly platform facing consumers with heterogeneous preferences for app quality, which prevents the platform from fully capturing the benefits from better apps. They show that the platform charges too high prices on both sides, yielding insufficient app quality and limited consumer participation. Furthermore, capping the app commission would enhance the app base but prompt the platform to raise its consumer price, to such an extent that consumer surplus would be reduced. We consider instead platform competition in a classic competitive bottlenecks setting, in which consumers have homogeneous preferences (at least ex ante), which enables the platform to appropriate the benefits from additional apps. We moreover focus on the volume of the app bases, which leads us to consider differences in development costs rather than in quality. As long as platforms compete for consumers, the commission maximizing their profit also maximizes consumer surplus (but exceeds the level maximizing total welfare); yet, competition leads to higher commissions whenever app development exhibits economies of scope.

Roadmap. We describe the general setting in Section 2. In Section 3, we adopt a stylized approach to present our key insights. In Section 4, we illustrate them in the context of a fully specified model with horizontally differentiated platforms and either joint or independent app development decisions. We study extensions in Section 5, and discuss policy implications in Section 6.

[11] Furthermore, in a previous version, we found that a complete choke-off no longer arises when platforms charge wholesale prices instead of ad valorem commissions on the app side.

2 Setting

Two platforms, $\mathcal{P}_1$ and $\mathcal{P}_2$, compete for (single-homing) consumers and (single- or multihoming) apps. On the app side, platforms set ad valorem commissions; app developers, facing heterogeneous investment costs, then decide on which platform(s) to invest, if any. On the consumer side, platforms set access prices; consumers then choose which platform to join, if any. This setting corresponds for example to the two leading mobile OS platforms (iOS and Android, with their app stores, App Store and Google Play), interpreting consumer prices as the prices of the devices (iPhone or Android phone), and treating for simplicity the Android platform as vertically integrated, like the iPhone platform.

We now present the model in more detail.

• *Consumers.* There is a continuum of consumers, each endowed with a stochastic value v for each app, drawn (independently across consumers and apps) from a distribution with c.d.f. $G(\cdot)$ over $\mathbb{R}_+$ and observed only after joining a platform. Let

$$d(p) \equiv 1 - G(p) \quad \text{and} \quad s(p) \equiv \int_p^{+\infty} d(\hat{p})\, d\hat{p}$$

denote consumers' expected demand and surplus from an app offered at price p. The resulting profit, $\pi(p) \equiv pd(p)$, is assumed to be maximal for some price p^m.

Each consumer also enjoys a platform-specific intrinsic utility u_i, for $i = 1, 2$. Hence, if $\mathcal{P}_i$ charges a price p_i and attracts y_i apps, each offering an expected surplus s_i, then the net payoff from joining $\mathcal{P}_i$ is[12]

$$u_i + s_i y_i - p_i = u_i - P_i,$$

where

$$P_i \equiv p_i - s_i y_i$$

denotes $\mathcal{P}_i$'s "quality-adjusted" price. The intrinsic utilities are distributed in such a way that the demand for $\mathcal{P}_i$ is[13]

$$D(P_i, P_j) > 0,$$

which features (imperfect) substitution: $\partial_2 D(\cdot) > 0$.

[12]For simplicity, each app belongs to a different category and consumers only care about their number. More generally, quality and diversity could matter as well, both within and across categories.

[13]The analysis readily extends to the case where $D(\cdot) = 0$ for high enough prices, with $\partial_2 D(\cdot) > 0$ and Assumption 1 below holding whenever $D(\cdot) > 0$.

• *Developers.* Apps being digital goods, their only costs are fixed investment costs,[14] which vary across developers and platforms. Specifically, each developer faces three costs: $k_i \geq 0$ for developing its app on $\mathcal{P}_i$, for $i = 1, 2$, and k for developing it on both platforms. Each cost realization $\boldsymbol{k} = (k_1, k_2, k)$ is independently drawn across developers from a distribution $\bar{F}(\boldsymbol{k})$ over $\mathbb{R}^3_+$, which is symmetric in k_1 and k_2.

If each $\mathcal{P}_i$ charges a commission a_i and provides access to a consumer base D_i, then offering the app on $\mathcal{P}_i$ at price $\tilde{p}_i$ gives the developer a payoff equal to

$$(1 - a_i)\,\pi(\tilde{p}_i)D_i - k_i,$$

whereas the payoff from offering the app on both platforms is given by:

$$(1 - a_1)\,\pi(\tilde{p}_1)D_1 + (1 - a_2)\,\pi(\tilde{p}_2)D_2 - k.$$

• *Platforms.* For the sake of exposition, we set the cost of servicing consumers to zero.[15] Hence, if each $\mathcal{P}_i$ charges a commission a_i and a consumer price p_i, and attracts y_i developers, each generating an expected profit π_i and consumer surplus s_i, then $\mathcal{P}_i$'s profit is given by, for $i \neq j \in \{1, 2\}$:

$$\Pi_i = (p_i + a_i\pi_i y_i)\, D\,(p_i - s_i y_i, p_j - s_j y_j)\,. \tag{1}$$

Without loss of generality, we focus on commissions not exceeding 1:[16]

$$a_i \in \mathcal{A} \equiv (-\infty, 1] \text{ for } i = 1, 2.$$

• *Timing.* The timing is as follows:

1. *Competition for apps*:

   (a) the two platforms set their commissions, a_1 and a_2;

   (b) developers learn their costs and make investment decisions.[17]

2. *Competition for consumers*:

[14]The analysis is robust to the introduction of variable costs, with the caveat that the commissions then also affect consumers through the price of the apps. See Remark 1.

[15]Alternatively, the price p_i can be interpreted as $\mathcal{P}_i$'s margin on the consumer side – to allow for this interpretation, we do not impose any restriction on the sign of p_i.

[16]Any higher commission deters any app development and is thus equivalent to a commission of 1. Relatedly, any derivative with respect to a_i must be understood as a left-hand derivative when evaluated at the boundary $a_i = 1$.

[17]Without loss of generality, developers join every platform for which they developed their apps.

(a) the platforms set their prices, p_1 and p_2, and developers set app prices;[18]

(b) consumers make their participation decisions; upon joining a platform, they learn their valuations for the available apps and decide which ones to buy.

At each stage, all decisions are simultaneous and public; hence, each stage determines a proper subgame. We will therefore focus on subgame-perfect equilibria.

As is well-known, multi-sided markets are subject to network effects and thus prone to tipping; as a result, competition – even between equally efficient firms – may lead to monopolization. We are instead interested here in the impact of competition on app development. We will therefore ignore tipping and focus on shared-market, symmetric equilibria.

3 A stylized approach

We first adopt a *stylized approach* and assume that, for any commissions set in stage $1a$, there exists a well-behaved continuation equilibrium in the key next stages, namely, the app development stage $1b$ and the platform pricing stage $2a$.

Using backward induction, we first consider the last stages of the game. In stage $2b$, consumers' participation decisions generate the demand $D\left(P_i, P_j\right)$. In stage $2a$, all developers charge the price p^m – regardless of the commissions;[19] each available app thus generates an expected profit $\pi^m \equiv \pi(p^m)$ per consumer, and each consumer obtains an expected surplus $s^m \equiv s(p^m)$ per app. For each $\mathcal{P}_i$, choosing a price p_i thus amounts to choosing a quality-adjusted price $P_i = p_i - s^m y_i$ and the resulting profit, given by (1), can be expressed as:

$$\Pi_i = \Pi\left(P_i, P_j; \sigma_i\right) \equiv \left(P_i + \sigma_i\right) D\left(P_i, P_j\right),$$

where

$$\sigma_i \equiv \left(s^m + a_i \pi^m\right) y_i.$$

It follows that, for any commissions (a_1, a_2) and app bases (y_1, y_2), the continuation subgame amounts to a classic price competition game, in which each $\mathcal{P}_i$ chooses a quality-adjusted price P_i and faces the demand $D\left(P_i, P_j\right)$, with the caveat that it benefits from a *subsidy* σ_i. In line with our stylized approach, we will suppose that, for any given subsidies σ_1 and σ_2, this game has a unique price equilibrium, in which

[18] Whether a multihoming developer can charge platform-specific prices does not affect the analysis.

[19] Indeed, $p^m = \arg\max_p\{(1-a_i)\pi(p)D_i\}$ for any $a_i < 1$ (and any $D_i > 0$). In the boundary case where $a_i = 1$, developers obtain zero profit and are thus indifferent about their pricing decisions; for the sake of exposition, we assume that they still charge p^m.

$\mathcal{P}_i$'s price is given by

$$P_i = P^e(\sigma_i, \sigma_j).$$

Let

$$\Pi^e(\sigma_i, \sigma_j) \equiv \Pi(P^e(\sigma_i, \sigma_j), P^e(\sigma_j, \sigma_i); \sigma_i) > 0 \tag{2}$$

denote $\mathcal{P}_i$'s equilibrium profit.[20] Intuitively, an increase in σ_i should benefit $\mathcal{P}_i$, but also induce it to price more aggressively (i.e., charge a lower quality-adjusted price), thus harming the rival. We will therefore maintain the following assumption:

Assumption 1 (competition for consumers) *For any $\sigma \in \mathbb{R}$ and:*

(a) $\partial_1 P^e(\sigma, \sigma) < \partial_2 P^e(\sigma, \sigma) < 0$;

(b) $\partial_1 \Pi^e(\sigma, \sigma) \geq -\partial_2 \Pi^e(\sigma, \sigma) > 0$.

Part (a) of Assumption 1 asserts that increasing one platform's subsidy reduces both quality-adjusted prices, and more so for the platform than for its rival. Part (b) asserts that such an increase benefits the platform but harms its rival, although to a lesser extent – it thus (weakly) enhances the platforms' joint profit.[21]

• *Competition for apps.* In stage $1b$, given the commissions (a_1, a_2) set in stage $1a$, developers base their investment decisions on expected consumer bases, which in turn depend on app development, and thus on the two commissions. Sticking to our stylized approach, we will assume that the distribution $\bar{F}(\boldsymbol{k})$ generates a unique, stable continuation equilibrium,[22] in which $\mathcal{P}_i$'s app base is given by

$$y^*(a_i, a_j),$$

which satisfies

$$y^*(0,0) > y^*(1,1) = 0.$$

The resulting subsidy for $\mathcal{P}_i$ is then given by

$$\sigma^*(a_i, a_j) \equiv (s^m + a_i \pi^m) y^*(a_i, a_j), \tag{3}$$

and therefore satisfies:

$$\partial_2 \sigma^*(a_i, a_j) = (s^m + a_i \pi^m) \partial_2 y^*(a_i, a_j). \tag{4}$$

[20]That this profit is positive follows from $D(\cdot) > 0$. If we allow for $D(\cdot) = 0$, the assumption $\Pi^e(\sigma_i, \sigma_j) > 0$ only needs to hold for non-negative subsidies.

[21]These assumptions are satisfied by common models of Bertrand competition featuring strategic complementarity, equilibrium stability and partial pass-through.

[22]Stability refers here to the usual robustness to small shocks in app bases – see proof of Lemma 3.

Hence, as long as $s^m + a_i\pi^m > 0$,[23] $\partial_2\sigma^*$ has the same sign as $\partial_2 y^*$: raising its rival's commission reduces $\mathcal{P}_i$'s subsidy if and only if it reduces $\mathcal{P}_i$'s app base.

Summing up, our stylized approach postulates the existence of a price function $P^e(\sigma_1, \sigma_2)$ (with associated profit $\Pi^e(\sigma_1, \sigma_2)$ given by (2)) satisfying Assumption 1 and of an app base function $y^*(a_i, a_j)$, such that, for any given commissions (a_1, a_2), there is a unique, stable continuation equilibrium, in which:

- in stage $1b$, $\mathcal{P}_i$'s app base is $y_i = y^*(a_i, a_j)$, generating the subsidy $\sigma_i = \sigma^*(a_i, a_j)$ given by (3);
- in stage $2a$, $\mathcal{P}_i$ charges a quality-adjusted price

$$P_i = P^*(a_i, a_j) \equiv P^e(\sigma^*(a_i, a_j), \sigma^*(a_j, a_i)); \tag{5}$$

- in stage $2b$, $\mathcal{P}_i$'s consumer base is

$$D_i = D^*(a_i, a_j) \equiv D(P^*(a_i, a_j), P^*(a_j, a_i)), \tag{6}$$

  and its profit is therefore

$$\Pi_i = \Pi^*(a_i, a_j) \equiv \Pi^e(\sigma^*(a_i, a_j), \sigma^*(a_j, a_i)). \tag{7}$$

3.1 Benchmarks

We first characterize the optimal commission a that a regulator would impose in stage $1a$,[24] given the continuation equilibria described above. We will denote by

$$\hat{y}(a) \equiv y^*(a, a) \quad \text{and} \quad \hat{\sigma}(a) \equiv \sigma^*(a, a) = (s^m + a\pi^m)\hat{y}(a)$$

the resulting app base and subsidy, and by

$$\hat{P}(a) \equiv P^e(\hat{\sigma}(a), \hat{\sigma}(a)) \quad \text{and} \quad \hat{D}(a) \equiv D(\hat{P}(a), \hat{P}(a))$$

the resulting quality-adjusted price and demand.

We distinguish two cases, depending on whether the regulator focuses on consumer surplus or social welfare.

[23] This is indeed the case for the commissions that maximize consumer surplus – see Corollary 1.

[24] For the sake of exposition, we focus on symmetric commissions, which is natural given the symmetry of the setting; moreover, the regulator may be constrained by non-discrimination provisions. The uniqueness of the continuation equilibria (for every $a \in \mathcal{A}$) implies that they are also symmetric.

3.1.1 Consumer surplus

Suppose first that the regulator seeks to maximize consumer surplus, given by:[25]

$$\hat{S}(a) \equiv \int_{\hat{P}(a)}^{+\infty} 2D(P,P)\, dP. \tag{8}$$

This amounts to minimizing the quality-adjusted price $\hat{P}(a)$. From Assumption 1(a), this in turn amounts to maximizing the subsidy $\hat{\sigma}(a)$. Furthermore, from Assumption 1(b), doing so also maximizes the joint profit of the platforms, given by

$$\hat{\Pi}_P(a) \equiv 2\Pi^e(\hat{\sigma}(a), \hat{\sigma}(a)). \tag{9}$$

The interests of platforms and consumers are therefore *aligned*:

Lemma 1 (consumer surplus) *Maximizing consumer surplus, $\hat{S}(a)$, or platforms' profit, $\hat{\Pi}_P(a)$, amounts to maximizing platforms' subsidy, $\hat{\sigma}(a)$. It follows that the commission that maximizes consumer surplus, a^S, satisfies $\hat{y}(a^S) > 0 > \hat{y}'(a^S)$ and:*

$$s^m + a^S \pi^m = \frac{\pi^m \hat{y}(a^S)}{-\hat{y}'(a^S)} > 0. \tag{10}$$

Proof. See Appendix A.1. ■

For ease of exposition, we will assume that a^S is uniquely characterized by (10).[26]

3.1.2 Social welfare

Suppose now that the regulator seeks to maximize social welfare, defined as the sum of all users' surplus and the platforms' profit:

$$\hat{W}(a) \equiv \hat{S}(a) + \hat{\Pi}_D(a) + \hat{\Pi}_P(a). \tag{11}$$

The first and last terms (consumer surplus and platforms' profit) are given by (8) and (9). The second term, representing developers' profit, can be expressed as:

$$\hat{\Pi}_D(a) \equiv \int_{\mathbb{R}_+^3} \pi_D(\hat{r}(a), \mathbf{k})\, d\bar{F}(\mathbf{k}), \tag{12}$$

where

$$\hat{r}(a) \equiv (1-a)\pi^m \hat{D}(a)$$

[25]For $P_1 = P_2 = P$, total demand is $2D(P,P) = 1 - H(P)$, where $H(\tilde{u})$ denotes the distribution of the maximal intrinsic value $\tilde{u} \equiv \max\{u_1, u_2\}$, and consumer surplus is given by $\tilde{S}(P) = \int_P^{+\infty} (\tilde{u} - P)\, dH(\tilde{u})$, which satisfies $\tilde{S}'(P) = -[1 - H(P)] = -2D(P,P)$.

[26]In case of multiple solutions, Lemma 2 holds for any of them, including the lowest one.

denotes the revenue that a developer can obtain by joining a platform, and

$$\pi_D(r, \mathbf{k}) \equiv \max\{0, r - k_1, r - k_2, 2r - k\}$$

denotes the equilibrium profit of a developer with cost realization $\mathbf{k} = (k_1, k_2, k_3)$.

As noted above, the commission a^S, which maximizes the subsidy $\hat{\sigma}(a)$, maximizes $\hat{S}(a)$ and $\hat{\Pi}_P(a)$ as well. Developers favor instead lower commissions, implying that the welfare-maximizing commission lies below a^S:[27]

Lemma 2 (social welfare) *The commission that maximizes social welfare, a^W, is such that $a^W < a^S$, $\hat{y}(a^W) > \hat{y}(a^S)(> 0)$, and:*

$$s^m + a^W \pi^m = \frac{\hat{P}(a^W) + (s^m + \pi^m)\hat{y}(a^W)}{-\hat{y}'(a^W)} \frac{\hat{D}'(a^W)}{\hat{D}(a^W)} \geq 0. \tag{13}$$

Proof. See Appendix A.2. ■

It follows from (13) that subsidies are socially desirable (all the more when focusing on consumers):

Corollary 1 (non-negative subsidies) *We have: $\hat{\sigma}(a^S) > \hat{\sigma}(a^W) \geq 0$; furthermore, $\hat{\sigma}(a^W) > 0$ unless $\hat{D}'(a^W) = 0$, in which case $\hat{\sigma}(a^W) = s^m + a^W \pi^m = 0$.*

Proof. See Appendix A.3. ■

3.2 Platform competition

We now show that platform competition can generate excessively high commissions. To this end, we complete our stylized approach by assuming that, in stage 1a, the commission-setting game – with payoffs $\Pi^*(a_i, a_j)$ given by (7) – is "well-behaved":

Assumption 2 (competition for developers) *For any $a \in \mathcal{A}$:*

- ***(a)*** *$\Pi^*(\tilde{a}, a)$ is strictly quasi-concave in $\tilde{a}$ in the range $\tilde{a} \in \mathcal{A}$;*
- ***(b)*** *$R(a) \equiv \arg\max_{\tilde{a}} \Pi^*(\tilde{a}, a)$ is differentiable and has a unique fixed point, a^C, which satisfies $|R'(a^C)| < 1$.*

[27] The commissions affect developers' revenue, $\hat{r}(a) = (1 - a)\pi^m \hat{D}(a)$, both directly and indirectly through consumer participation $\hat{D}(a)$. However, a slight departure from a^S has only a second-order indirect effect on subsidies and consumer participation, which is thus dominated by the direct effect.

Part (a) ensures that the platforms have a unique best-response, $R(\cdot)$; part (b) ensures in turn that there exists a unique, locally stable equilibrium, in which $a_1 = a_2 = a^C$.

Our first proposition shows that the comparison between this equilibrium commission and what would maximize consumer surplus hinges on a simple condition:

Proposition 1 (platform competition) *Platform competition yields higher (resp., lower) commissions than those maximizing consumer surplus whenever raising one commission reduces (resp., increases) the rival's app base. Formally:*

$$a^C \gtreqless a^S \quad \textit{if and only if} \quad \partial_2 y^* \left(a^S, a^S\right) \lesseqgtr 0.$$

Proof. See Appendix A.4. ■

The key is therefore whether the platforms are complements or substitutes for the development of apps. In case of *complements*, that is, if raising one commission *reduces* the rival's app base, competition generates higher commissions than what would maximize consumer surplus – and welfare, as $a^S > a^W$. This is because a platform has an incentive to reduce the rival's subsidy to make the latter less aggressive. In case of *substitutes*, that is, if raising one commission *fosters* app development on the rival platform, competition generates lower commissions than those maximizing consumer surplus – they may however still exceed those maximizing social welfare.

Remark 1 (pass-through) *The analysis carries over when introducing variable costs for the apps, with the caveat that the commissions then affect app prices. For example, a unit cost $c > 0$ per user induces developers present on $\mathcal{P}_i$ to charge*

$$p_i = p^m(a_i) \equiv \arg\max_p \{(1 - a_i)(p - \frac{c}{1 - a_i})d(p)\},$$

which increases with a_i. Hence, a_i additionally affects $\mathcal{P}_i$'s subsidy σ_i through its impact on consumer surplus, $s(p^m(a_i))$, and on the profit generated by the apps, $\pi(p^m(a_i))$, which are now both decreasing in a_i. Yet, a_i affects $\mathcal{P}_j$'s subsidy σ_j only through the latter's app base, as before; Proposition 1 thus remains valid – maximizing platforms' subsidy (and, thus, consumer surplus and platforms' profit) may however call for lower commissions, to avoid depressing the surplus and profit generated by the apps.

3.3 Cost externalities

The above analysis highlights a key factor, namely, the impact of a commission on the rival platform's app base. We now show that this, in turn, depends on the existence of (dis-)economies of scope in app development.

3.3.1 Preliminaries

In stage 1, $\mathcal{P}_i$'s app base can be expressed as $y^*(a_i, a_j) = Y(r^*(a_i, a_j), r^*(a_j, a_i))$, for $i \neq j \in \{1, 2\}$, where

$$r^*(a_i, a_j) \equiv (1 - a_i)\, \pi^m D^*(a_i, a_j) \tag{14}$$

denote the revenue expected from joining $\mathcal{P}_i$, and

$$Y(r_i, r_j) \equiv \Pr\left[\max\{r_i - k_i, r_i + r_j - k\} \geq \max\{r_j - k_j, 0\}\right] \tag{15}$$

characterizes developers' decisions to invest on $\mathcal{P}_i$, given (the distribution of investment costs and) the revenues offered by the two platforms. The impact of a marginal increase in the rival's commission, a_j, can in turn be expressed as:

$$\partial_2 y^*(a_i, a_j) = \partial_1 Y(r^*(a_i, a_j), r^*(a_j, a_i))\, \partial_2 r^*(a_i, a_j) + \partial_2 Y(r^*(a_i, a_j), r^*(a_j, a_i))\, \partial_1 r^*(a_j, a_i).$$

Using (14) and evaluating at $a_i = a_j = a^S$ yields:

$$\partial_2 y^*\left(a^S, a^S\right) = \mathcal{D}^S + \mathcal{I}^S,$$

where, using $D^S \equiv D^*\left(a^S, a^S\right)$ and $r^S \equiv r^*\left(a^S, a^S\right)$,

$$\mathcal{D}^S \equiv -\pi^m D^S \partial_2 Y\left(r^S, r^S\right)$$

captures the impact of a change in the rival's commission, da_j, through its *direct* effect on the revenue offered by the rival (i.e., $dr_j = -\pi^m D^S da_j$), whereas

$$\mathcal{I}^S \equiv \left(1 - a^S\right) \pi^m \left[\partial_2 D^*\left(a^S, a^S\right) \partial_1 Y\left(r^S, r^S\right) + \partial_1 D^*\left(a^S, a^S\right) \partial_2 Y\left(r^S, r^S\right)\right]$$

reflects instead the impact of this change through its *indirect effects* on the revenues offered by the two platforms, as a result of the alterations of their consumer bases (i.e., $dr_h = (1 - a_h)\, \pi^m dD_h$, for $h = i, j$).

Let $P^S \equiv P^*\left(a^S, a^S\right)$, $\sigma^S \equiv \sigma^*\left(a^S, a^S\right)$, and

$$A^S \equiv \left[\partial_1 Y(\cdot) - \partial_2 Y(\cdot)\right]\left[\partial_1 D(\cdot) - \partial_2 D(\cdot)\right]\left[\partial_1 P^e(\cdot) - \partial_2 P^e(\cdot)\right]\left(1 - a^S\right) \pi^m \left(s^m + a^S \pi^m\right), \tag{16}$$

where, for $h = 1, 2$, $\partial_h Y(\cdot)$ is evaluated at $r_1 = r_2 = r^S$, $\partial_h D(\cdot)$ is evaluated at $P_1 = P_2 = P^S$, and $\partial_h P^e(\cdot)$ is evaluated at $\sigma_1 = \sigma_2 = \sigma^S$. We have:

Lemma 3 (direct effect) $A^S < 1$ *and*

$$\partial_2 y^* \left(a^S, a^S\right) = \frac{\mathcal{D}^S}{1 - A^S}. \tag{17}$$

Proof. See Appendix A.5. ■

The intuition is that the direct effect ($\mathcal{D}^S$) drives the indirect ones ($\mathcal{I}^S$). Specifically, $\mathcal{D}^S$ generates the initial impact on the rival's app base in stage 1, keeping consumer bases unchanged in stage 2. However, this initial impact alters platforms' subsidies and, therefore, their consumer prices and resulting consumer bases. The alteration of consumer bases triggers a first additional impact on the rival's app base, summarized by the factor A^S. This, in turn, generates a second iteration of indirect effects, and so on.[28]

Together with Proposition 1, Lemma 3 shows that whether competitive commissions lie above or below the level maximizing consumer surplus is driven by the sign of the direct effect, $\mathcal{D}^S$. The proof of the lemma rests on two legs. The first leg relies on the observation that consumer prices – and, thus, consumer bases – are driven by platforms' subsidies, which in turn are driven by their app bases. The second leg relies on the assumed stability of continuation equilibria, implying that it is robust to small changes in app bases.

3.3.2 Independent development

Following Armstrong (2006), the literature on competitive bottlenecks has mainly focused on the case of *simultaneous participation* decisions (i.e., both sides of the market decide at the same time) and *independent participation* decisions on the multihoming side (i.e., joining one platform has no incidence on the decision to join the other platform). To study how the insights from this literature apply to our setting with *sequential participation* decisions (developers deciding which platform to join, if any, before consumers' participation decisions), we consider here the particular case where k_1 and k_2 are symmetrically and independently distributed across developers, with marginal c.d.f. $F(\cdot)$, and:

$$k = k_1 + k_2.$$

This assumption eliminates any cross-platform externality between an individual developer's investment decisions. Specifically, developing the app for one platform does not affect the cost of developing it for the other platform; furthermore, individual apps being infinitesimal, this has no incidence either on consumers' subsequent participation decisions and, therefore, on the revenue offered by the other platform. Hence, as

[28] That is, we have $\mathcal{I}^S = A^S \mathcal{D}^S + (A^S)^2 \mathcal{D}^S + \ldots$, which leads to (17).

in Armstrong (2006), developers' participation decisions are made *independently* for each platform. In particular, $Y(\cdot)$ boils down to, for $i \neq j \in \{1, 2\}$:

$$Y(r_i, r_j) = F(r_i),$$

implying $\mathcal{D}^S = \left(-\pi^m D^S \partial_2 Y(r_i, r_j)\right) = 0$. Hence, we have:

Proposition 2 (independent development) *In case of independent development decisions, platform competition yields the commissions that maximize consumer surplus:* $a^C = a^S$.

Proof. It follows directly from Proposition 1, Lemma 3 and $\mathcal{D}^S = 0$. ■

The underlying intuition is as follows. With independent development decisions, the revenue offered by a platform has no direct impact on its rival's app base (i.e., $\mathcal{D}^S = 0$). This, in turn, implies that small deviations from the equilibrium commissions do not alter the consumer bases. To see why, suppose tentatively that, starting from $a_i = a_j = a^C$, a unilateral deviation in a_j has no indirect impact either on $\mathcal{P}_i$'s app base, which thus remains equal to $y^C \equiv y^*\left(a^C, a^C\right)$. It follows that the deviation has no impact on $\mathcal{P}_i$'s subsidy; that is:

$$\sigma_i = \sigma^*\left(a^C, a_j\right) = \left(s^m + a^C \pi^m\right) y^C = \sigma^C.$$

$\mathcal{P}_j$'s profit from the deviation is therefore equal to $\Pi(\sigma_j, \sigma_i) = \Pi\left(\sigma^*\left(a_j, a^C\right), \sigma^C\right)$; this, in turn, implies that $\mathcal{P}_j$'s equilibrium commission maximizes its own subsidy, $\sigma_j = \sigma^*\left(a_j, a^C\right)$; the deviation thus has no (first-order effect) on that subsidy. In other words, following the deviation, both subsidies remain equal to σ^C, which validates our working assumption: in stage 2, both consumer bases remain unaffected. Hence, there are indeed no indirect effects, and so $\partial_2 y^*\left(a^C, a^C\right) = 0$. It then follows from Proposition 1 that $a^C = a^S$.

It follows from these observations that, with independent development decisions, a^S and a^C both coincide with a^σ, the commission maximizing the platform's own subsidy (assuming that the rival platform charges that commission); that is, a^σ is characterized by the following condition:

$$a^\sigma = \arg\max_a \sigma^*(a, a^\sigma),$$

or:

$$\partial_1 \sigma^*(a^\sigma, a^\sigma) = 0. \tag{18}$$

Corollary 2 (independent development) *In case of independent development decisions, in equilibrium each platform seeks to maximize its own subsidy:* $a^C = \left(a^S =\right) a^\sigma$.

Proof. See Appendix A.6. ■

Remark 2 (competitive bottlenecks) *Proposition 2 extends the result of Armstrong (2006), established for environments in which (participation decisions are independent, and) platforms compete simultaneously on both sides of the market, to environments in which platforms compete on the multihoming side (here, the app side) before competing on the single-homing side (here, the consumer side). There is a twist, however. In Armstrong (2006), the equilibrium fee charged by a platform on the multihoming side maximizes the joint surplus of the platform and of its users on the single-homing side. Here, in case of independent development decisions, the equilibrium commissions maximize both total consumer surplus and the platforms' joint profits (and not only their sum).*[29]

Our insights carry over when platforms compete in wholesale prices for apps, but no longer do so when they compete in fixed fees (see Online appendix O-A). The reason is that, in our sequential competition setting, wholesale prices, like ad valorem commissions, affect platforms' profits only through the subsidies enjoyed when interacting with consumers; the interests of consumers and platforms are therefore aligned, as both seek to maximize these subsidies. In contrast, fixed fees have a direct impact on platforms' profits, other than through the subsidies. As a result, the interests of consumers and platforms diverge.

3.3.3 Economies of scope

We now show that the above insight does not carry over when developers face (dis-)economies of scope. Specifically, suppose that development costs are distributed as follows: k_1 and k_2 are symmetrically and independently distributed across developers, with marginal c.d.f. $F(\cdot)$, and

$$k = \kappa\left(k_1, k_2; s\right),$$

where $s \in \mathbb{R}$ is a parameter reflecting (dis-)economies of *scope* and $\kappa(\cdot)$ has the following properties:

[29]The underlying arguments are also somewhat different. In Armstrong (2006), starting from any given fees, a platform deviating on the multihoming side can capture the resulting change of surplus on the other side (by adjusting its fee on that other side). Here, deviating on the app side in stage 1 triggers instead a change in consumer surplus on both platforms in stage 2 (the "indirect effects" described above); however, starting from the *equilibrium* commissions, these indirect effects are only second-order.

Assumption S (economies of scope). $\kappa(k_1, k_2; s)$ is symmetric in k_1 and k_2, such that $\kappa(k_1, k_2; 0) = k_1 + k_2$, and strictly decreasing in s for any $(k_1, k_2) > (0, 0)$.

The boundary case $s = 0$ corresponds to independent development decisions (i.e., $k = k_1 + k_2$). By contrast, developers benefit from economies of scope (i.e., $k < k_1 + k_2$) if $s > 0$, and face diseconomies of scope (i.e., $k > k_1 + k_2$) if instead $s < 0$. Illustrative examples include $\kappa(k_1, k_2; s) = \exp(-s)(k_1 + k_2)$, $\kappa(k_1, k_2; s) = k_1 + k_2 - s$ and $\kappa(k_1, k_2; s) = k_1 + k_2 - sk_1k_2$.

The following proposition shows that platform competition yields higher (resp., lower) commissions than those maximizing consumer surplus whenever there are economies (resp., diseconomies) of scope:

Proposition 3 (economies of scope) *Under Assumption S:*

$$a^C \gtreqless a^S \quad \textit{if and only if} \quad s \gtreqless 0.$$

Proof. See Appendix A.7. ■

In the light of Proposition 1 and Lemma 3, it suffices to show that the direct effect $\mathcal{D}^S = -\pi^m D^S \partial_2 Y\left(r^S, r^S\right)$ is negative (resp., positive) when developers benefit from economies of scope (resp., face diseconomies of scope), that is:

$$\partial_2 Y\left(r^S, r^S\right) \gtreqless 0 \Longleftrightarrow s \gtreqless 0.$$

To study this, let us examine the effect of an increase in r_j on developers who are at the margin between developing or not their apps for $\mathcal{P}_i$. Those facing a very high k_j must be indifferent between developing for $\mathcal{P}_i$ only and not developing at all, and those facing a very low k_j must be indifferent between developing for both platforms and developing only for $\mathcal{P}_j$. The decisions of these developers do not depend on r_j: in the former case, r_j does not affect any of the two relevant options and, in the latter case, it affects both options in the same way.

Developers facing intermediate values of k_j are either indifferent between developing for both platforms or for none of them, or indifferent between developing for $\mathcal{P}_1$ only or for $\mathcal{P}_2$ only. The former case occurs when $r_1 + r_2 - k = 0 > \max\{r_1 - k_1, r_2 - k_2\}$; it can therefore arise only when there are economies of scope. Furthermore, raising r_j then increases $\mathcal{P}_i$'s app base: $\partial_2 Y(r_i, r_j) > 0$. The latter case occurs instead when $r_1 - k_1 = r_2 - k_2 > \max\{r_1 + r_2 - k, 0\}$, and thus arises only when there are diseconomies of scope; furthermore, raising r_j then decreases $\mathcal{P}_i$'s app base: $\partial_2 Y(r_i, r_j) < 0$.

Economies of scope arise whenever the cost of porting an app to another platform is lower than the initial cost of development, which we expect to be often the case in practice.[30] Conversely, whenever there are economies of scope, combining Lemma 2 and Proposition 3 yields:

Corollary 3 *When there are economies of scope, we have:*

$$a^C > a^S > a^W.$$

Although we have focused on development costs, a similar logic can be applied to operational costs or demand synergies (e.g., if multihoming is required to reach a viable scale or generate demand). The recent complaint of the U.S. Department of Justice against Apple provides several examples. For instance, providers of food delivery or ride-sharing services need to develop their apps for both Android phones and iPhones to reach a viable scale.[31] Likewise, social or money-sharing apps must enable users on Android devices to interact with users on iPhones and vice versa.[32] In some cases, the restrictions imposed by Apple on certain features (besides the commission) has prompted additional concerns for the development of intrinsically-multihoming apps. For example, according to the U.S. Department of Justice, some U.S. banks have abandoned the development of digital-wallet apps. Another company decided not to offer an innovative digital car key because Apple required it to add its features into Apple Wallet rather than solely in its own app.[33]

4 Illustration

We now illustrate the above insights using a classic horizontal differentiation setting on the consumer side, and two distinct types of developers on the app side.

• *Consumers.* The two platforms are located at the two ends of a unit-length Hotelling segment, along which consumers are uniformly distributed. Upon joining a platform, a consumer obtains an intrinsic utility $u_0 > 0$ and faces a transportation cost $t > 0$ per unit of distance. As in Armstrong (1998) and Laffont, Rey and Tirole (1998a,b), u_0 is supposed to be large enough to ensure that all consumers join a platform (full participation). Consumers know their locations before joining a platform and, from the above, anticipate an expected surplus s^m from each app present on the platform. Hence, if $\mathcal{P}_i$ charges consumers a price p_i and attracts y_i apps, joining that platform

[30] This is particularly true for cloud-gaming apps, as the game is played on cloud servers and, thus, need not be compatible with different mobile OSs.

[31] U.S. DOJ (2024), at §160.

[32] U.S. DOJ (2024), at §161.

[33] U.S. DOJ (2024), at §132.

gives a consumer located at distance x_i a net payoff equal to:

$$u_0 + s^m y_i - p_i - tx.$$

The consumer indifferent between the two platforms is located at a distance x from $\mathcal{P}_i$ (and, thus a distance $1-x$ from $\mathcal{P}_j$, for $i \neq j \in \{1,2\}$) equal to:

$$\frac{1}{2} + \frac{s^m(y_i - y_j) - (p_i - p_j)}{2t}.$$

Using the quality-adjusted price $P_i = p_i - s^m y_i$, the demand for $\mathcal{P}_i$ is therefore:

$$D(P_i, P_j) = \frac{1}{2} - \frac{P_i - P_j}{2t}.$$

As before, $\mathcal{P}_i$'s profit is equal to $\Pi_i = (P_i + \sigma_i) D(P_i, P_j)$, where $\sigma_i = (s^m + a_i \pi^m) y_i$. In stage 2, the platforms compete à la Hotelling, which leads to:

Lemma 4 (Hotelling competition) *In stage 2, for any given (σ_1, σ_2), competition for consumers leads to $P^e(\sigma_i, \sigma_j) = P^H(\sigma_i, \sigma_j)$ and $\Pi^e(\sigma_i, \sigma_j) = \Pi^H(\sigma_i, \sigma_j)$, where*

$$P^H(\sigma_i, \sigma_j) \equiv t - \frac{2\sigma_i + \sigma_j}{3} \quad \text{and} \quad \Pi^H(\sigma_i, \sigma_j) \equiv \frac{1}{2t}\left(t + \frac{\sigma_i - \sigma_j}{3}\right)^2.$$

Proof. See Appendix B.1. ■

It can be seen from Lemma 4 that Assumption 1 is satisfied.[34]

• *Developers.* Some apps are made available on multiple platforms whereas others are developed only for a specific platform. To capture this in a simple way, we distinguish two types of app developers: a fraction $\alpha \in [0,1]$ of them are intrinsic multihomers in that they develop their apps on either both platforms or none, whereas the others make independent development decisions for each platform – as will become clear, whether the same or different developers make those decisions is immaterial for the analysis; see Remark 3 below.

Specifically, the latter type of developers (*independent decision makers* hereafter) face platform-specific costs k_1 and k_2, symmetrically and independently distributed across developers, with marginal c.d.f. $F_I(\cdot)$ and density $f_I(\cdot) > 0$ over $[0, \infty)$; hence, as in Section 3.3.2, developing the app on one platform has no incidence on the development decision for the other platform.[35] The former type of developers (*joint decision*

[34] Specifically, $\partial_1 P^e(\cdot) = 2\partial_2 P^e(\cdot) = -2/3$ and $\partial_1 \Pi^e(\sigma, \sigma) = -\partial_2 \Pi^e(\sigma, \sigma) = 1/3$.

[35] The case $\alpha = 0$, in which all developers make independent decisions, corresponds to the setting of Etro (2023) (in which investment costs are moreover independently distributed across platforms).

makers hereafter) can instead make their apps available on both platforms at cost k, drawn from a distribution with c.d.f. $F_J(\cdot)$ and density $f_J(\cdot) > 0$ over $[0, \infty)$.[36] It is worth noting that we are agnostic about the relative strength of the distributions $F_I(\cdot)$ and $F_J(\cdot)$; by contrast, the proportion α of joint decision makers will play an important role.

Remark 3 (interpretation) *This cost distribution is a particular case of the distribution $\bar{F}(\mathbf{k})$ introduced in Section 2, in which joint decision makers benefit from substantial economies of scope (i.e., $k \ll k_1 + k_2$), whereas independent decision makers have no such benefits (i.e., $k = k_1 + k_2$). Alternatively, an independent decision maker can be interpreted as a pair of specialized developers, each dedicated to a distinct platform (with $k_i \ll k, k_j$ for those dedicated to $\mathcal{P}_i$). The case $\alpha = 1$ can also reflect a situation in which the operating systems of the two platforms are fully interoperable (e.g., due to regulation or thanks to third-party facilitating tools), so that the cost of porting an app from one platform to the other is zero (hence, $k = \min\{k_1, k_2\}$).*

To ensure the existence of a well-behaved equilibrium under competition, we will maintain the following assumption:

Assumption 3 (illustration) *The density functions are non-increasing:*

$$f_I'(\cdot) \leq 0 \quad \text{and} \quad f_J'(\cdot) \leq 0.$$

4.1 Benchmarks

If both platforms charge the same commission a,[37] in the symmetric continuation equilibrium a developer can obtain $(1-a)\,\pi^m$ when multihoming, and half of it when single-homing; hence, there are

$$\hat{y}\,(a) = (1-\alpha)\hat{y}_I(a) + \alpha\hat{y}_J(a) \tag{19}$$

apps available on each platform, where $\hat{y}_I(a)$ and $\hat{y}_J(a)$ are the proportions of active developers in the two groups, and are given by:

[36] One interpretation is that these developers can costlessly port their apps from one platform to the other. Introducing a small cost of porting these apps would not qualitatively affect the analysis.

[37] When commissions are symmetric, there always exists a unique symmetric equilibrium, in which developers expect each platform to attract half of the consumers and, as a result, the proportions of active developers are given by (20) and (21). This moreover constitutes the unique continuation equilibrium if t is large enough. For the sake of exposition, we focus on this continuation equilibrium.

$$\hat{y}_I(a) \equiv F_I\left(\frac{1-a}{2}\pi^m\right), \quad (20)$$

$$\hat{y}_J(a) \equiv F_J\left((1-a)\pi^m\right). \quad (21)$$

By construction, the relative importance of the two groups, measured by α, affects the total number of apps available on a platform; by contrast, it has no direct impact on $\hat{y}_I(a)$ nor $\hat{y}_J(a)$ – as we will see, it has an indirect impact through the level of the commissions. The resulting subsidy is

$$\hat{\sigma}(a) \equiv (s^m + a\pi^m)\,\hat{y}(a), \quad (22)$$

leading to the (quality-adjusted) price and profit:

$$\hat{P}(a) \equiv P^H(\hat{\sigma}(a), \hat{\sigma}(a)) = t - \hat{\sigma}(a)$$

and

$$\hat{\Pi} \equiv \Pi^H(\hat{\sigma}(a), \hat{\sigma}(a)) = \frac{t}{2}.$$

Competition thus induces the platforms to pass on their subsidy entirely to consumers; as a result, their profits do not depend on the commission.

As total consumer demand is inelastic (namely, $2\hat{D}(a) = 1$), it follows from Lemma 2 that the welfare-maximizing commission a^W nullifies the subsidy: $s^m + a^W\pi^m = 0$, implying $\hat{\sigma}(a^W) = 0$; that is, regardless of the proportion α of joint decision makers and of the differentiation parameter t, the social planner subsidizes the developers so as to align their profit per consumer, $(1-a^W)\pi^m$, with the total surplus generated by their apps, $s^m + \pi^m$.

By contrast, the commission that maximizes consumer surplus, a^S, always generates a positive subsidy, as noted by Corollary 1. Furthermore, from Lemma 4, for symmetric commissions, the platforms' equilibrium consumer bases are equal to one-half, regardless of t. Hence, the revenue they offer to developers and, therefore, the resulting app bases and subsidies, are all independent of t. It follows that a^S, which from Lemma 1 seeks to maximizes platforms' subsidies, is also independent of t.

Building on this leads to:

Lemma 5 (illustration - benchmarks)

(i) a^W and a^S satisfy:

$$a^W = -\frac{s^m}{\pi^m} < a^S(\alpha) < 1 \text{ and } \hat{\sigma}\left(a^W\right) = 0 < \hat{\sigma}\left(a^S(\alpha)\right).$$

(ii) Neither a^W nor a^S depends on t.

Proof. See Appendix B.2. ■

Example 1 (uniform distribution) *When development costs are uniformly distributed over $[0,1]$ (i.e., $F_I(k) = F_J(k) = k$), a^S is also independent of α and equal to:*

$$a^S = \frac{\pi^m - s^m}{2\pi^m} (\in (a^W, \frac{1}{2})). \tag{23}$$

In particular, $a^S \lesseqgtr 0$ if and only if $s^m \gtreqless \pi^m$.

Remark 4 (monopoly profit) *We already noted that the commission a^S maximizes the profit of the platforms when they compete for consumers. Interestingly, in the Hotelling setting, $a = a^S$ also maximizes the monopoly profit that an integrated firm, operating both platforms, could obtain when exploiting consumers. This is because total demand, being here inelastic, is therefore the same under competition and monopoly (as long as full participation remains optimal). It follows that the app bases generated by a commission a are also the same in both situations. As a monopolist can appropriate the consumer value generated by the apps, it then finds it optimal to maximize $\hat{\sigma}(a)$.*[38]

4.2 Platform competition

We now show that platform competition leads indeed to excessively high commissions. As in Section 3, for any given commissions (a_1, a_2) set in stage 1a, let $y^*(a_i, a_j)$ denote $\mathcal{P}_i$'s app base in the continuation equilibrium, and $\sigma^*(a_i, a_j)$ denote its subsidy, given by (3). Building on Lemma 4, $\mathcal{P}_i$'s expected demand satisfies

$$D^*(a_i, a_j) = \frac{1}{2} + \frac{\Delta^*(a_i, a_j)}{6t}, \tag{24}$$

where $\Delta^*(a_i, a_j) \equiv \sigma^*(a_i, a_j) - \sigma^*(a_j, a_i)$ denotes $\mathcal{P}_i$'s *subsidy advantage*, and the revenue from joining $\mathcal{P}_i$ is given by:

$$r^*(a_i, a_j) = (1 - a_i)\,\pi^m D^*(a_i, a_j). \tag{25}$$

[38] Given the unit demand and the absence of operating costs, the monopoly profit, which corresponds to the sum of the monopoly price on the consumer side and the platforms' revenue from apps, is equal to $u_0 - t/2 + \hat{\sigma}(a)$, and is thus maximal for a^S.

Furthermore, $\mathcal{P}_i$'s app base is $y^*(a_i, a_j) = (1-\alpha)y_I^*(a_i, a_j) + \alpha y_J^*(a_i, a_j)$, where:

$$y_I^*(a_i, a_j) \equiv F_I(r^*(a_i, a_j)), \tag{26}$$
$$y_J^*(a_1, a_2) \equiv F_J(r^*(a_1, a_2) + r^*(a_2, a_1)). \tag{27}$$

Finally, the subsidy advantage can be expressed as

$$\Delta^*(a_i, a_j) = s^m[y_I^*(a_i, a_j) - y_I^*(a_j, a_i)] + \pi^m[a_i y^*(a_i, a_j) - a_j y^*(a_j, a_i)] \tag{28}$$

Together, equations (24) to (28) jointly characterize the continuation equilibrium. Furthermore, from Lemma 4, in equilibrium each platform seeks to maximize its subsidy advantage, given by (28).

The following proposition establishes the existence of a unique equilibrium and highlights its key features:

Proposition 4 (illustration) *For t large enough, there exists a unique equilibrium, which is symmetric. Furthermore, whenever a symmetric equilibrium exists, it is unique and the resulting commission, a^C:*

- (i) *does not depend on t.*
- (ii) *is strictly increasing in α and:*
  - *coincides with a^S for $\alpha = 0$;*
  - *strictly exceeds a^S for $\alpha > 0$;*
  - *is equal to 1 (thus choking off the development of apps) for $\alpha = 1$.*

Proof. See Appendix B.3. ■

When all development decisions are taken independently for each platform (i.e., for $\alpha = 0$), as in Etro (2023) and Section 3.3.2, competition induces the platforms to adopt the commissions that maximize consumer surplus. This no longer holds in the presence of joint decision makers who, as in Section 3.3.3, benefit from economies of scope. The rival's subsidy is then

$$\sigma_j = (s^m + a_j\pi^m)[(1-\alpha)y_I^*(a_j, a_i) + \alpha y_J^*(a_j, a_i)],$$

where the number of apps stemming from joint decision making, $y_J^*(a_j, a_i)$, is now directly affected by $\mathcal{P}_i$'s own commission. Each platform then has an additional incentive to raise its commission, as reducing the number of these apps decreases its rival's subsidy – all the more so when α is large.

In particular, when all developing decisions are jointly made (i.e., $\alpha = 1$), platforms' incentives lead them to *choke off* entirely the development of apps: $a^C(1) = 1$, leading to $y_1 = y_2 = 0$ – indeed, in that case, there cannot be any symmetric or asymmetric equilibrium without choke-off.[39] Interestingly, there is no longer a complete choke-off when platforms compete in wholesale prices rather than ad valorem commissions; this is because wholesale prices generate double-marginalization problems, which in turn act as a disciplining device.[40]

Recall that the commission that maximizes consumer surplus is always higher than the welfare-maximizing level; thus, for $\alpha = 0$ we have:

$$a^C = a^S > a^W,$$

and for any $\alpha > 0$, we have:

$$a^C > a^S > a^W.$$

The following example illustrates these insights.

Example: uniform distribution. When development costs are uniformly distributed over $[0, 1]$ (i.e., $F_I(k) = F_J(k) = k$), the equilibrium commission is equal to:

$$a^C(\alpha) \equiv a^S + \alpha \frac{\pi^m + s^m}{2\pi^m},$$

which strictly increases from a^S to 1 as α increases from 0 to 1. It follows that each platform's app base, given by

$$\hat{y}(a^C(\alpha)) = \frac{1 - \alpha^2}{4}(\pi^m + s^m),$$

strictly decreases to 0 as α increases. By contrast, as a^S is independent of α (see (23)), the app base maximizing consumer surplus, $\hat{y}(a^S)$, *increases* with α. Figure 1 presents the values of interest, for the commission and the resulting app base.

[39] To see this, it suffices to note that $\mathcal{P}_i$'s subsidy advantage becomes $\Delta^*(a_i, a_j) = (a_i - a_j) y_J^*(a_1, a_2) \pi^m$. It follows that, starting from $a_i < a_j$, say, $\mathcal{P}_i$ would have an incentive to match its rival's commission; and starting from $a_1 = a_2 = a$, both platforms would have an incentive to raise their commissions as long as their remains some app development, as $\partial_1 \Delta^*(a, a) = y_J^*(a, a) \pi^m$.

[40] Llobet and Padilla (2016) consider a setting in which upstream firms license their patents to a downstream firm. They compare ad valorem royalties with per-unit royalties and show that the former bring two welfare benefits. First, ad valorem royalties make double marginalization less severe, not only between the downstream firm and its partners but also among upstream firms (the so-called royalty stacking problem), which leads to lower prices. This, in turn, gives all firms greater incentives to invest in complementary technologies. In our paper, ad valorem commissions actually eliminate double marginalization entirely, because applications are digital goods with zero marginal costs. Interestingly, it is exactly this absence of double marginalization which harms consumers by choking off app development, thereby overturning the welfare comparison between ad valorem commissions and per-unit royalties.

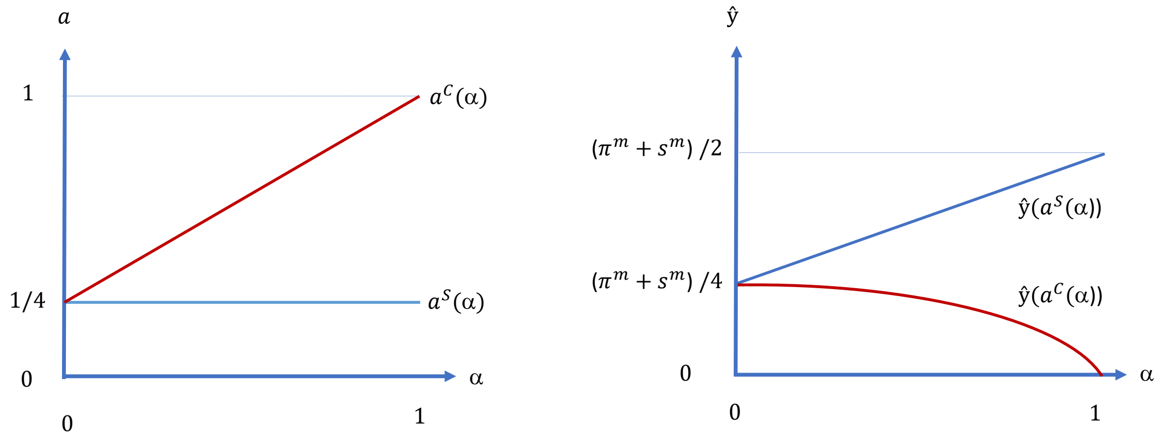

Figure 1: Commission and app bases for uniform distributions: $F_I(k) = F_J(k) = k$ and $\pi^m = 2s^m$.

The finding that a higher fraction of joint decision makers, who benefit from economies of scope, raises the complementarity between the two platforms and induces them to charge higher commissions, is consistent with the following claim of the U.S. Department of Justice:

"...the vast majority of developers consider iPhones and Android devices as complements because developers must build apps that run on both platforms due to the lack of user multi-homing. ... This market reality increases the power that Apple is able to exercise over developers that seek to reach users on smartphones."[41]

Remark 5 (interoperability) *As alreadny noted, the case $\alpha = 1$ can be interpreted as the platforms' operating systems being fully interoperable. For exogenously given commissions, moving from no or partial interoperability to full interoperability generates economies of scope and thereby boosts app development. In contrast, our analysis shows that when commissions are endogenous, full interoperability can hinder and even choke off app development.*

5 Extensions

In this section, we first provide an extension of the Hotelling model in which $n \geq 2$ platforms compete. We show that an increase in the number of platforms amplifies the importance of platform complementarity for the app development, which in turn widens the gap between the equilibrium commission and the consumer-surplus maximizing one. We also examine a variant of the Hotelling model that incorporates the uncertainty of app success and the sequential nature of the development and porting

[41] U.S. Department of Justice (2024), at § 181.

decisions. We provide a condition under which app development exhibits platform complementarity.

5.1 Multiple platforms

We here extend the Hotelling model to the spokes model (Caminal and Claici, 2007, Chen and Riordan, 2007),[42] with $n \geq 2$ symmetric spokes and n symmetric platforms. The main objective of this extension is to examine how the number of platforms affects the gap between the equilibrium commission and the consumer-surplus maximizing one.

All spokes are identical: they have an origin ($x = 0$), a length normalized to 1/2, and they all join at the centre of the market ($x = 1/2$). Each $\mathcal{P}_i$ is located at the origin of spoke i, and a unit mass of consumers is uniformly distributed over the n spokes. A consumer located on spoke i is only interested in $\mathcal{P}_i$ and another, randomly selected platform – every other platform being selected with equal probability. As before, a consumer incurs a disutility proportional to the distance between her and the platform; we let $t > 0$ denote the transportation parameter. Finally, the intrinsic value from buying a device, u_0, is supposed to be high enough to ensure that the market is covered. For $n = 2$, this is the Hotelling model of Section 4.

For $i = 1, ..., n$, let $P_i = p_i - s^m y_i$ and $\sigma_i = (s^m + a_i \pi^m) y_i$ denote as before $\mathcal{P}_i$'s quality-adjusted price and subsidy, and

$$\bar{P}_i \equiv \frac{1}{n-1} \sum_{j \neq i} P_j \text{ and } \bar{\sigma}_i \equiv \frac{1}{n-1} \sum_{j \neq i} \sigma_j$$

denote the average price and subsidy of its rivals; the demand for $\mathcal{P}_i$ and its profit are now given by (with the superscript S referring to the Spokes model):

$$\bar{D}_n^S \left(P_i, \bar{P}_i\right) \equiv \frac{1}{n} - \frac{P_i - \bar{P}_i}{nt} \text{ and } \bar{\Pi}_n^S \left(P_i, \bar{P}_i; \sigma_i\right) \equiv \left(P_i + \sigma_i\right) \bar{D}_n^S \left(P_i, \bar{P}_i\right).$$

In stage 2, platform competition leads here to:

Lemma 6 (Spokes competition) *In stage 2, for any given subsidies* $(\sigma_1, \cdots, \sigma_n)$, *competition for consumers leads to* $P_i = P_n^S(\sigma_i, \bar{\sigma}_i)$, $D_i = D_n^S(\Delta_i)$ *and* $\Pi_i = \Pi_n^S(\Delta_i) \equiv nt\left[D_n^S(\Delta_i)\right]^2$, *where* $\Delta_i = \sigma_i - \bar{\sigma}_i$ *denotes* $\mathcal{P}_i$*'s subsidy advantage and:*

$$P_n^S(\sigma_i, \bar{\sigma}_i) \equiv t - \frac{n\sigma_i + (n-1)\bar{\sigma}_i}{2n-1}, \quad \textit{and} \quad D_n^S(\Delta_i) \equiv \frac{1}{n} + \frac{n-1}{2n-1}\frac{\Delta_i}{nt}. \tag{29}$$

Proof. See Online Appendix O-B.1.1. ■

[42] For a recent overview of the properties of this model, see Reggiani (2020).

Lemma 6 extends Lemma 4 to more than 2 platforms. A platform's profit depends again on its subsidy advantage, compared to the average subsidy of its rivals.

On the app side, we maintain the assumption that a fraction α of developers are joint decision makers, who can develop their apps on all platforms at a cost distributed according to F_J, whereas the others face platform-specific development costs, symmetrically and independently distributed across developers, with marginal c.d.f. F_I; we also maintain Assumption 3. If all platforms charge the same commission a, in the symmetric continuation equilibrium each platform attracts a fraction $1/n$ of consumers and $\hat{y}_n^S(a) = \alpha \hat{y}_J(a) + (1-\alpha)\hat{y}_I^S(a,n)$ apps, where the proportion of active joint decision makers remains given by (21), whereas for independent decision makers it becomes:

$$\hat{y}_I^S(a,n) \equiv F_I\left(\frac{1-a}{n}\pi^m\right). \tag{30}$$

The resulting subsidy is $\hat{\sigma}_n^S(a) \equiv (s^m + a\pi^m)\,\hat{y}_n^S(a)$, and there is again full pass-through: the (quality-adjusted) price and profit are $\hat{P}_n^S(a) \equiv P_n^S\left(\hat{\sigma}_n^S(a), \hat{\sigma}_n^S(a)\right) = t - \hat{\sigma}_n^S(a)$ and $\hat{\Pi}_n^S(a) \equiv \Pi_n^S(0) = t/n$. As before, maximizing consumer surplus amounts to maximizing the subsidy $\hat{\sigma}_n^S(a)$, and platforms' profit is again independent of the commission. Maximizing social welfare takes into account developers' profit, which can be expressed as $\hat{\Pi}_D^S(a,n) \equiv (1-a)\,\pi^m \hat{y}_n^S(a) - \hat{K}_n^S(a)$, where the total investment cost is equal to:

$$\hat{K}_n^S(a) \equiv \alpha \int_0^{(1-a)\pi^m} k dF_J(k) + (1-\alpha)\, n \int_0^{(1-a)\frac{\pi^m}{n}} k dF_I(k).$$

Building on this leads to:

Lemma 7 (multiple platforms - benchmarks)

(*i*) *The commission that maximizes social welfare does not depend on the number of platforms; it thus remains equal to* $a^W = -s^m/\pi^m\,(<0)$, *regardless of* n, α *or* t, *and generates zero subsidy:* $\hat{\sigma}_n^S\left(a^W\right) = 0$.

(*ii*) *The commission that maximizes consumer surplus (and, thus, platforms' subsidy),* $a_n^S(\alpha)$, *does not depend on* t*; it moreover lies strictly between* a^W *and* 1, *and generates a positive subsidy:* $\hat{\sigma}_n^S\left(a_n^S(\alpha)\right) > 0$*; furthermore, for any* α *in* $[0,1]$, $a_n^S(\alpha)$ *is bounded away from* 1 *as* n *goes to infinity.*

Proof. See Online Appendix O-B.1.2. ■

Remark 6 (monopoly profit) *As total demand remains inelastic, Remark 4 extends to more than 2 platforms: the commission a^S also maximizes the monopoly profit that an integrated firm, operating all platforms, could obtain when exploiting consumers.*

From Lemma 6, in stage 1 each $\mathcal{P}_i$ chooses its commission so as to maximize its subsidy advantage Δ_i, as in the Hotelling duopoly setting, and a marginal deviation has again only a second-order effect on platforms' consumer bases. Building on these observations leads to:

Proposition 5 (multiple platforms) *For t large enough, there exists a unique equilibrium, which is symmetric. Furthermore, whenever a symmetric equilibrium exists, it is unique and the equilibrium commission, $a_n^C(\alpha)$:*

(i) is independent of t and increasing in α;

(ii) tends to 1, thus choking-off development of apps, as n goes to infinity.

Proof. See Online Appendix O-B.1.3. ■

The proposition shows that what matters is not the intensity of platform competition, reflected in t, but the number of platforms, n. Increasing this number reduces each platform's share of consumers, which in turn tilts the balance in favor of joint development. This has therefore the same effect as an increase in the proportion of joint decision makers, α: increasing n or α exacerbates the importance of platform complementarity and thus widens the gap between the equilibrium commission, $a_n^C(\alpha)$, and the consumer-surplus maximizing one, $a_n^S(\alpha)$. Furthermore, in both instances, pushing the logic to its limit chokes-off the development of apps: $a_n^C(\alpha)$ tends to 1 as n goes to infinity or α tends to 1. These insights are in line with the situation observed in the Chinese smartphone market, in which five large competing manufacturers, each controlling its own app store, charge a 50 percent commission to app developers.

Example 1 (cont'd) *We show in Online Appendix O-B.1.3 that, when development costs are uniformly distributed over $[0,1]$ (i.e., $F_I(k) = F_J(k) = k$), the commission that maximizes consumer surplus is independent of n (and thus remains given by (23), which does not depend on α either); platform competition yields instead:*

$$a_n^C(\alpha) \equiv \frac{\pi^m - s^m - (n-1)\alpha\pi^m}{2\pi^m + (n-2)\alpha\pi^m} = 1 - \frac{(1-\alpha)\pi^m + s^m}{2\pi^m + (n-2)\alpha\pi^m},$$

which strictly increases with n for any given $\alpha > 0$.

5.2 Sequential development

In practice, app development is a risky venture, as the success of apps is uncertain. To mitigate this risk, developers often develop their apps for one platform first, and then port it onto other platforms if sufficiently successful. We note here that our insights carry over when accounting for this possibility.

To see why, suppose that (i) apps are popular with some probability and of no value otherwise, and (ii) porting an app only costs a small fraction of the initial development investment. It readily follows that popular apps are all ported, regardless of the platform for which they were initially developed. As a result, both platforms attract the same relevant apps; an increase in either commission has therefore a negative impact on the rival platform's app base.

We show in Online Appendix O-B.2 that this remains the case as long as (i) unsuccessful apps are sufficiently less valuable than popular ones, and (ii) the cost of porting an app on a given platform is sufficiently lower than the cost of developing the app from scratch for that platform. Specifically, we adapt the Hotelling setting of Section 4 by supposing that an app is popular with probability $\lambda \in (0,1)$. If popular, it generates as before a revenue $r_i = (1-a_i)\pi^m D_i$ on $\mathcal{P}_i$. Otherwise, only a fraction $\eta \in [0,1)$ of consumers find it interesting; it thus generates a revenue ηr_i and an expected surplus ηs^m.[43] The success of an app is idiosyncratic and independent of the platform for which it is initially developed. We assume further that, for $i \neq j \in \{1,2\}$, each app can be developed for $\mathcal{P}_i$ at cost k_i and then ported onto $\mathcal{P}_j$ at cost δk_j, where k_1 and k_2 are independently drawn from the same distribution over $[\underline{k}, \bar{k}]$, and $\delta \in (0,1)$. Finally, we suppose that:

$$\eta < \frac{2\delta \underline{k}}{s^m + \pi^m} \, (<) \, \frac{2\underline{k}}{s^m + \pi^m} < \frac{2\lambda}{1+\lambda}.$$

The first inequality ensures that unsuccessful apps are never ported, whereas the last one ensures that apps with low enough costs are developed.[44]

In a symmetric equilibrium, each platform offers a revenue $r^C \equiv (1-a^C)\pi^m/2$ to successful apps. Hence, developing an app for $\mathcal{P}_i$ yields an expected profit equal to $\rho^C - k_i$, where

$$\rho^C \equiv [\lambda + (1-\lambda)\eta] \, r^C,$$

[43] We maintain the assumption that consumers learn their app values after joining a platform.

[44] In equilibrium, platforms charge $a > -s^m/\pi^m$ to obtain a positive subsidy; hence, the revenue generated by unsuccessful apps, $\eta(1-a)\pi^m/2$, cannot exceed $\eta(s^m+\pi^m)/2$; the first condition implies that this cannot cover the lowest possible cost of porting an app, $\delta\underline{k}$. The last condition ensures instead that charging a close enough to $-s^m/\pi^m$ generates a positive app base: developers with (k_1, k_2) close to $(\underline{k}, \underline{k})$ then find it profitable to develop their app and port it if popular. For any λ, s^m, π^m and δ, both conditions hold for low enough $\underline{k}$ and η.

whereas developing it for $\mathcal{P}_i$ and porting it when successful generates a profit $\rho^C - k_i + \lambda\left(r^C - \delta k_j\right)$. Hence, if a developer faces $k_i < k_j$, for $i \neq j \in \{1,2\}$, then (see Figure 2):

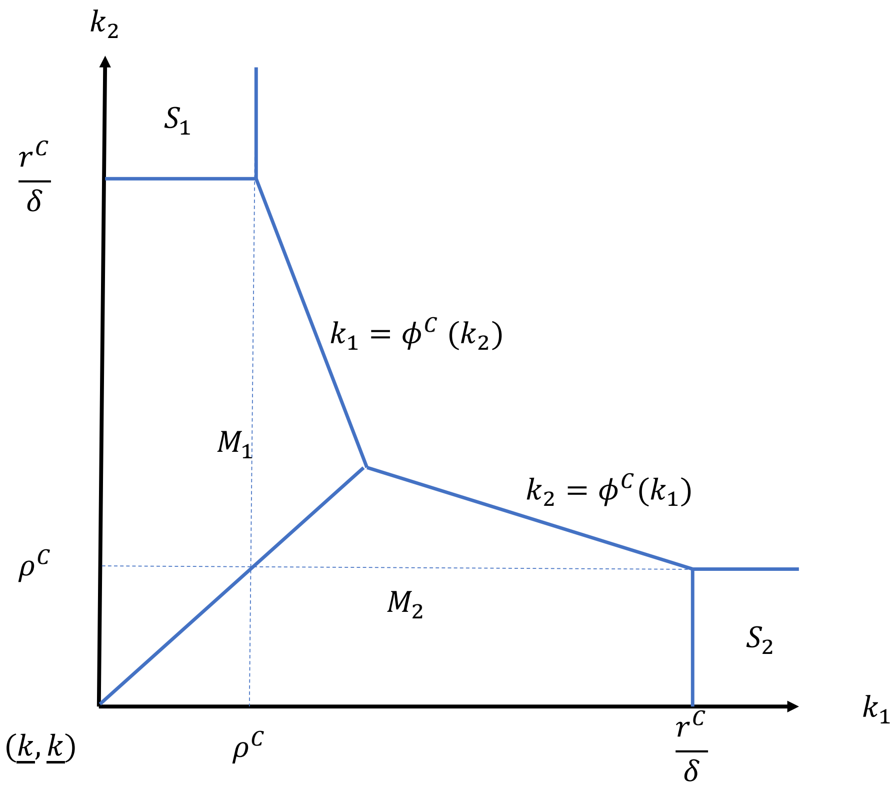

Figure 2: App development and porting decisions.

- if $\delta k_j > r^C$ and $k_i < \rho^C$ (region S_i), then the app is developed for $\mathcal{P}_i$ but not ported, regardless of its success (single-homing on $\mathcal{P}_i$);
- if instead $\delta k_j < r^C$ and $k_i < \phi^C(k_j)$ (region M_i), where

$$\phi^C(k) \equiv \rho^C + \lambda\left(r^C - \delta k\right),$$

  then the app is developed for $\mathcal{P}_i$ and ported when successful (possibly multihoming, starting on $\mathcal{P}_i$);
- in all other cases, the app is not developed.

We show in Online Appendix O-B.2 that an increase in $\mathcal{P}_i$'s commission affects its rival's weighted app base (where the weights are respectively equal to $\lambda + (1-\lambda)\eta$ for the region $M_j \cup S_j$, λ for the region M_i and 0 everywhere else) through its impact on the expected revenue from "developing and porting when popular". On the one hand, reducing this expected revenue induces some developers to drop out: the region $M_i \cup M_j$ thus shrinks, which adversely affects $\mathcal{P}_j$'s app base (by a factor λ for M_i and $\lambda + (1 - \lambda)\,\eta$ for M_j). On the other hand, it induces other developers to switch from $\mathcal{P}_i$ to $\mathcal{P}_j$ as initial development platform: the region M_j thus grows at the expense of M_i, which expands $\mathcal{P}_j$'s app base by a factor $(1 - \lambda)\,\eta$.[45] It follows that raising $\mathcal{P}_i$'s commission reduces $\mathcal{P}_j$'s weighted app base whenever the revenue generated by unsuccessful apps is small enough (i.e., η low enough), implying that the popular apps – which benefit from economies of scope – play a large role.

6 Policy implications

The debate on the commissions charged to developers has centered on the effectiveness of platform competition; on the one hand, Apple and Google argue that competition is intense and acts as a disciplining device;[46] on the other hand, regulators argue that commissions are excessive because, in practice, competition is limited due to consumer preferences and biases, as well as to switching costs and entrenchment strategies.[47] Our analysis proposes a drastically different perspective,[48] in that *platform competition* may be the *source* of the problem, rather than a cure.

Specifically, in Section 5.1, we find that increasing the number of competing platforms leads to even *higher* commissions, rather than lower ones; and in Sections 4 and 5.1, the degree of *substitution* between platforms has no incidence whatsoever on the commissions. These insights suggest that a policy intervention designed to foster platform competition is unlikely to have the desired impact on the commissions charged to developers.

Our analysis also highlights the role of economies of scope and joint development. In Section 3.3.3, competition generates commissions that exceed the level maximizing consumer surplus whenever there are economies of scope; in Section 5.2, the same occurs whenever popular apps (i.e., those benefiting from economies of scope) play a key role – as appears to be the case in practice. In the same vein, in Sections 4 and 5.1, the equilibrium commissions increase with the proportion of joint development. This

[45]Increasing $\mathcal{P}_i$'s commission also reduces the revenue from "developing for $\mathcal{P}_i$ and never porting" (the region S_i thus shrinks), and induces some developers to switch from "developing for $\mathcal{P}_j$ and porting if successful" to "developing for $\mathcal{P}_j$ and never porting" (the region S_j thus expands at the expense of M_j). However, these effects have no impact on $\mathcal{P}_j$'s app base.

[46]For instance, Apple argued the 30 percent commission was determined in competitive conditions in 2008 and has not increased since then (U.K. CMA, 2022, p. 133). See also the responses of Apple and Google to the interim report of the CMA(2021).

[47]See, e.g., U.K. CMA (2022) at p. 138: "Overall, we consider that the lack of competition faced by the App Store and Play Store allows them to charge above a competitive rate of commission to app developers".

[48]We thank Jorge Padilla for this comment.

suggests that policy measures designed to foster *interoperability* may actually exacerbate the problem rather than solve it – indeed, in the case of perfect interoperability, all developers become joint decision makers.

In contrast, encouraging competition between multiple app stores on the same platform, or side-loading (i.e., allowing app downloads from third-party websites) may constitute promising avenues,[49] provided that the access conditions for rival app stores or side-loaded apps are properly regulated and enforced.[50]

[49]It is worth noting that the Digital Markets Act aims at promoting interoperability as well as app store competition and side-loading.

[50]The experience of South Korea highlights the need to supervise access conditions. In response to a ban on the exclusivity of their integrated payment systems, Apple and Google each imposed a 26% licensing fee on developers opting for third-party payment systems; as a result, the ban had no significant impact on app development – see Chang and Miller (2024). For a recent analysis of desirable access conditions, see Bisceglia and Tirole (2023).

Appendix

A Stylized approach

A.1 Proof of Lemma 1

We already established that maximizing consumer surplus or platforms' profit amounts to maximizing the subsidy $\hat{\sigma}(a)$. By revealed preferences, we then have:

$$\left(s^m + a^S \pi^m\right) \hat{y}\left(a^S\right) = \hat{\sigma}\left(a^S\right) \geq \hat{\sigma}(0) = s^m \hat{y}(0) > 0,$$

where the last inequality stems from $\hat{y}(0) = y^*(0,0) > 0$. As by construction $\hat{y}(\cdot) \geq 0$, it follows that $\hat{y}\left(a^S\right) > 0$. The solution is therefore interior and the first-order condition leads to (10).

A.2 Proof of Lemma 2

The following lemmas will be useful:

Lemma A.1 *App developers' profit, $\hat{\Pi}_D(a)$, and the number of apps on each platform, $\hat{y}(a)$, both vary like $\hat{r}(a)$: for any $a, a' \leq 1$, $\hat{\Pi}_D(a) > \hat{\Pi}_D(a') \Leftrightarrow \hat{y}(a) > \hat{y}(a') \Leftrightarrow \hat{r}(a) > \hat{r}(a')$.*

Proof. The result follows from that, in equilibrium, a developer's expected profit is given by $\pi_D(\hat{r}(a), \mathbf{k})$, which is increasing in $\hat{r}(a)$ in the range $\hat{r}(a) \geq 0$ (that is, for $a \leq 1$), and strictly so for the marginal developers (i.e., those for which $\max\{\hat{r}(a) - k_1, \hat{r}(a) - k_2, 2\hat{r}(a) - k\} = 0$) as well as for the infra-marginal ones (i.e., those for which $\max\{\hat{r}(a) - k_1, \hat{r}(a) - k_2, 2\hat{r}(a) - k\} > 0$). Hence, an increase in $\hat{r}(a)$: (i) increases the number of apps present on a platform, $\hat{y}(a)$, by inducing marginal developers to develop their apps; and (ii) also increases developers' total profit, $\hat{\Pi}_D(a)$, both by increasing the number of apps present on the platforms, and by increasing the individual profit generated by each such app. ∎

Lemma A.2 *We have:*

(i) $\hat{\sigma}'(a^W) > 0$ *and* $\hat{D}'\left(a^W\right) \geq 0$*;*

(ii) $\hat{y}'\left(a^W\right) < 0$.

Proof. Part *(i)*. Recall that the quality-adjusted price, $\hat{P}(a)$, is decreasing in the subsidy $\hat{\sigma}(a)$; hence, the consumer demand, $\hat{D}(a)$, is weakly increasing in the subsidy $\hat{\sigma}(a)$. Therefore, if $\hat{\sigma}'(a^W) \leq 0$, then a marginal reduction in a from a^W would:

- weakly increase consumer surplus and platforms' profit, from Lemma 1;
- strictly increase developers' revenue share, $1-a$, and weakly increase consumer demand $\hat{D}(a)$; hence, this would therefore strictly increase developers' revenue, $\hat{r}(a)$, and thus, from Lemma A.1, developers' profit, $\hat{\Pi}_P(a)$.

It follows that that a marginal reduction in a from a^W would strictly enhance social welfare, a contradiction. Therefore, $\hat{\sigma}'(a^W) > 0$, which in turn implies $\hat{D}'\left(a^W\right) \geq 0$.

Part (ii). It follows from $\hat{\sigma}'(a^W) > 0$ that a marginal increase in a from a^W would strictly increase consumer surplus and platforms' profit. As social welfare is maximal for $a = a^W$, it must be the case that such a marginal increase in a would strictly reduce developers' profit; hence, from Lemma A.1, $\hat{y}'\left(a^W\right) < 0$. ■

Total welfare, $\hat{W}(a)$, is given by (11). Hence, $\hat{W}'(a) = \hat{S}'(a) + \hat{\Pi}'_P(a) + \hat{\Pi}'_D(a)$, where:

$$\begin{aligned} \hat{S}'(a) + \hat{\Pi}'_P(a) &= -2\hat{D}(a)\hat{P}'(a) + 2\left\{\left[\hat{P}'(a) + \hat{\sigma}'(a)\right]\hat{D}(a) + \left[\hat{P}(a) + \hat{\sigma}(a)\right]\hat{D}'(a)\right\} \\ &= 2\left\{\hat{\sigma}'(a)\hat{D}(a) + \left[\hat{P}(a) + \hat{\sigma}(a)\right]\hat{D}'(a)\right\} \end{aligned} \quad (A.1)$$

and:[51]

$$\hat{\Pi}'_D(a) = 2\hat{r}'(a)\hat{y}(a) = -2\pi^m\hat{y}(a)\hat{D}(a) + 2(1-a)\pi^m\hat{y}(a)\hat{D}'(a). \quad (A.2)$$

From Lemma 1, consumer surplus, $\hat{S}(a)$, platforms' profit, $\hat{\Pi}_P(a)$, and platforms' subsidy, $\hat{\sigma}(a) = (s^m + a\pi^m)\hat{y}(a)$, are all maximal for $a = a^S$; hence:

$$\begin{aligned} S\left(a^S\right) + \Pi_P\left(a^S\right) &\geq S\left(a^W\right) + \Pi_P\left(a^W\right), \quad (A.3) \\ \left(s^m + a^S\pi^m\right)\hat{y}\left(a^S\right) &\geq \left(s^m + a^W\pi^m\right)\hat{y}\left(a^W\right), \quad (A.4) \end{aligned}$$

and:

$$\hat{S}'\left(a^S\right) = \hat{\Pi}'_P\left(a^S\right) = \hat{D}'\left(a^S\right) = 0. \quad (A.5)$$

Together, ($A.2$) and ($A.5$) lead to:

$$\hat{W}'\left(a^S\right) = \hat{\Pi}'_D\left(a^S\right) = -2\pi^m\hat{y}\left(a^S\right)\hat{D}\left(a^S\right) < 0,$$

where the inequality follows from $\hat{y}\left(a^S\right) > 0$ and $\hat{D}\left(a^S\right) > 0$ (from Lemma 1). It follows that $\hat{W}\left(a^W\right) > \hat{W}\left(a^S\right)$ (as starting from $a = a^S$, a slight reduction in a would

[51] The expression reflects the negative impact of an increase in a on the revenue $\hat{r}(a)$ of the $\hat{y}(a)$ developers present on each platform. Raising a also induces some marginal developers to drop out, and may also induce some marginal multihomers to switch to single-homing; however, these additional impacts have zero first-order effect.

increase welfare, which is maximal for $a = a^W$), that is:

$$S\left(a^W\right) + \Pi_P\left(a^W\right) + \Pi_D\left(a^W\right) > S\left(a^S\right) + \Pi_P\left(a^S\right) + \Pi_D\left(a^S\right). \tag{A.6}$$

Combining (A.3) and (A.6) yields:

$$\Pi_D\left(a^W\right) > \Pi_D\left(a^S\right).$$

It then follows from Lemmas 1 and A.1 that:

$$\hat{r}\left(a^W\right) > \hat{r}\left(a^S\right) > 0, \tag{A.7}$$

$$\hat{y}\left(a^W\right) > \hat{y}\left(a^S\right) > 0. \tag{A.8}$$

It moreover follows from (A.7) that:

$$a^W < 1. \tag{A.9}$$

Furthermore, combining (A.4) and (A.8) yields:

$$s^m + a^W \pi^m \leq \left(s^m + a^S \pi^m\right) \frac{\hat{y}\left(a^S\right)}{\hat{y}\left(a^W\right)} < s^m + a^S \pi^m,$$

implying:

$$a^W < a^S.$$

Finally, it follows from $\hat{y}\left(a^W\right) > 0$ that the social optimum is interior; the welfare-maximizing commission is thus characterized by the first-order condition $\hat{W}'\left(a^W\right) = 0$, which, using (A.1) and (A.2), amounts to:

$$\left(s^m + a^W \pi^m\right) \hat{y}'\left(a^W\right) \hat{D}\left(a^W\right) = -\left[\hat{P}\left(a^W\right) + \hat{\sigma}\left(a^W\right) + \left(1 - a^W\right) \pi^m \hat{y}\left(a^W\right)\right] \hat{D}'\left(a^W\right).$$

Dividing by $-\hat{y}'\left(a^W\right) \hat{D}\left(a^W\right)$ (where $\hat{y}'\left(a^W\right) < 0$ from Lemma A.2, and $\hat{D}\left(\cdot\right) > 0$ by assumption) then leads to:

$$s^m + a^W \pi^m = \frac{\hat{P}\left(a^W\right) + \hat{\sigma}\left(a^W\right) + \left(1 - a^W\right) \pi^m \hat{y}\left(a^W\right)}{-\hat{y}'\left(a^W\right)} \frac{\hat{D}'\left(a^W\right)}{\hat{D}\left(a^W\right)},$$

where:

- from Lemma A.2, $-\hat{y}'\left(a^W\right) \hat{D}\left(a^W\right) > 0$ (as just noted) and $\hat{D}'\left(a^W\right) \geq 0$;
- $\hat{P}\left(a^W\right) + \hat{\sigma}\left(a^W\right) > 0$ (as platforms' profit is positive from Assumption 1) and $\left(1 - a^W\right) \pi^m \hat{y}\left(a^W\right) > 0$ (as $\hat{y}\left(a^W\right) > 0$ (from (A.8)) and $a^W < 1$ (from (A.9)).

Hence, $s^m + a^W \pi^m \geq 0$, with a strict inequality unless $\hat{D}'\left(a^W\right) = 0$.

A.3 Proof of Corollary 1

From Lemma A.2, $\hat{\sigma}'(a^W) > 0$. It follows that $\hat{\sigma}\left(a^S\right) > \hat{\sigma}\left(a^W\right)$ (as starting from $a = a^W$, a slight increase in a would increase the subsidy, which is maximal for $a = a^S$). Furthermore, $\hat{y}\left(a^W\right) > 0$ from (A.8) and, from the end of the proof of Lemma 2, $s^m + a^W \pi^m = 0$ if $\hat{D}'\left(a^W\right) = 0$, and $s^m + a^W \pi^m > 0$ otherwise. It follows that $\hat{\sigma}\left(a^W\right) \geq 0$, with a strict inequality unless $\hat{D}'\left(a^W\right) = 0$.

A.4 Proof of Proposition 1

Assumption 2(b) (namely, equilibrium uniqueness and local stability) implies that $a^* > a^S$ if and only $R\left(a^S\right) > a^S$; Assumption 2(a) (namely, strict quasi-concavity) ensures in turn that $R\left(a^S\right) > a^S$ if and only if $\partial_1 \Pi^*\left(a^S, a^S\right) > 0$. Furthermore (with $\sigma^S \equiv \sigma^*\left(a^S, a^S\right)$):

$$\begin{aligned}
\partial_1 \Pi^*\left(a^S, a^S\right) &= \partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) \partial_1 \sigma^*\left(a^S, a^S\right) + \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right) \partial_2 \sigma^*\left(a^S, a^S\right) \\
&= -\left[\partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) - \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right)\right] \partial_2 \sigma^*\left(a^S, a^S\right) \\
&= -\left[\partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) - \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right)\right] \left(s^m + a^S \pi^m\right) \partial_2 y^*\left(a^S, a^S\right),
\end{aligned}$$

where the second equality stems from the first-order condition

$$0 = \hat{\sigma}'\left(a^S\right) = \partial_1 \sigma^*\left(a^S, a^S\right) + \partial_2 \sigma^*\left(a^S, a^S\right),$$

and the last equality follows from (4). From Assumption 1(b), $\partial_1 \Pi^e\left(\sigma^S, \sigma^S\right) > \partial_2 \Pi^e\left(\sigma^S, \sigma^S\right)$. It follows that $\partial_1 \Pi^*\left(a^S, a^S\right) > 0$ if and only if $\partial_2 y^*\left(a^S, a^S\right) < 0$, which concludes the argument. A similar reasoning establishes that $a^* < a^S$ (resp., $a^* = a^S$) if and only if $\partial_2 y^*\left(a^S, a^S\right) > 0$ (resp., $\partial_2 y^*\left(a^S, a^S\right) = 0$).

A.5 Proof of Lemma 3

The proof relies on two claims.

Claim A.1 (direct effect) $\mathcal{I}^S$ *satisfies:*

$$\mathcal{I}^S = A^S \partial_2 y^*\left(a^S, a^S\right).$$

Proof. By construction, for $a_i = a_j = a^S$ we have:

$$\partial_1 \sigma^* \left(a^S, a^S\right) + \partial_2 \sigma^* \left(a^S, a^S\right) = 0, \tag{A.10}$$

which in turn implies:

$$\partial_1 P^* \left(a^S, a^S\right) + \partial_2 P^* \left(a^S, a^S\right) = \left[\partial_1 P^e(\cdot) + \partial_2 P^e(\cdot)\right] \left[\partial_1 \sigma^* \left(a^S, a^S\right) + \partial_2 \sigma^* \left(a^S, a^S\right)\right] = 0, \tag{A.11}$$

where $\partial_i P^e(\cdot)$ is evaluated at $\sigma_1 = \sigma_2 = \sigma^S$. Likewise:

$$\partial_1 D^* \left(a^S, a^S\right) + \partial_2 D^* \left(a^S, a^S\right) = \left[\partial_1 D(\cdot) + \partial_2 D(\cdot)\right] \left[\partial_1 P^* \left(a^S, a^S\right) + \partial_2 P^* \left(a^S, a^S\right)\right] = 0, \tag{A.12}$$

where $\partial_i D(\cdot)$ is evaluated at $P_1 = P_2 = P^S$.

Furthermore, differentiating (5) and using (A.10) yields:

$$\partial_2 P^* \left(a^S, a^S\right) = \partial_1 P^e(\cdot)\, \partial_2 \sigma^* \left(a^S, a^S\right) + \partial_2 P^e(\cdot)\, \partial_1 \sigma^* \left(a^S, a^S\right) = \left[\partial_1 P^e(\cdot) - \partial_2 P^e(\cdot)\right] \partial_2 \sigma^* \left(a^S, a^S\right). \tag{A.13}$$

Similarly, differentiating (6) leads to:

$$\begin{aligned} \partial_2 D^* \left(a^S, a^S\right) &= \partial_1 D(\cdot)\, \partial_2 P^* \left(a^S, a^S\right) + \partial_2 D(\cdot)\, \partial_1 P^* \left(a^S, a^S\right) \\ &= \left[\partial_1 D(\cdot) - \partial_2 D(\cdot)\right] \partial_2 P^* \left(a^S, a^S\right) \\ &= \left[\partial_1 D(\cdot) - \partial_2 D(\cdot)\right] \left[\partial_1 P^e(\cdot) - \partial_2 P^e(\cdot)\right] \left(s^m + a^S \pi^m\right) \partial_2 y^* \left(a^S, a^S\right), \end{aligned} \tag{A.14}$$

where the second equality stems from (A.11) and the last one from (A.13) and (4).

We thus have (with $\partial_1 Y(\cdot)$ evaluated at $r_1 = r_2 = r^S$):

$$\begin{aligned} \mathcal{I}^S &= \left(1 - a^S\right) \pi^m \left[\partial_2 D^* \left(a^S, a^S\right) \partial_1 Y(\cdot) + \partial_1 D^* \left(a^S, a^S\right) \partial_2 Y(\cdot)\right] \\ &= \left(1 - a^S\right) \pi^m \left[\partial_1 Y(\cdot) - \partial_2 Y(\cdot)\right] \partial_2 D^* \left(a^S, a^S\right) \\ &= A^S \partial_2 y^* \left(a^S, a^S\right), \end{aligned}$$

where the second equality is from (A.12) and the last one is from (A.14) and (16). ■

Claim A.2 (stability) $A^S < 1$.

Proof. Suppose that, starting from $a_1 = a_2 = a^S$, $y_1 = y_2 = y^S$, $\sigma_1 = \sigma_2 = \sigma^S$, $P_1 = P_2 = P^S$, $D_1 = D_2 = D^S \equiv D^* \left(a^S, a^S\right)$, and $r_1 = r_2 = r^S$, there is an exogenous transfer of dy from y_j to y_i (i.e., $dy_i = -dy_j = dy > 0$), and consider the implications for the app and customer bases. We have:

$$\begin{aligned} d\sigma_i &= \left(s^m + a^S\pi^m\right) dy_i = d\sigma \equiv \left(s^m + a\pi^m\right) dy, \\ d\sigma_j &= \left(s^m + a\pi^m\right) dy_j = -\left(s^m + a\pi^m\right) dy = -d\sigma, \end{aligned}$$

$$\begin{aligned} dP_i &= \partial_1 P^e\left(\sigma^S,\sigma^S\right) d\sigma_i + \partial_2 P^e\left(\sigma^S,\sigma^S\right) d\sigma_j = dP \equiv \left[\partial_1 P^e\left(\sigma^S,\sigma^S\right) - \partial_2 P^e\left(\sigma^S,\sigma^S\right)\right] d\sigma, \\ dP_j &= \partial_2 P^e\left(\sigma^S,\sigma^S\right) d\sigma - \partial_1 P^e\left(\sigma^S,\sigma^S\right) d\sigma = -dP, \end{aligned}$$

$$\begin{aligned} dD_i &= \partial_1 D\left(P^S,P^S\right) dP_i + \partial_2 D\left(P^S,P^S\right) dP_j = dD \equiv \left[\partial_1 D\left(P^S,P^S\right) - \partial_2 D\left(P^S,P^S\right)\right] dP, \\ dD_j &= \partial_2 D\left(P^S,P^S\right) dP - \partial_1 D\left(P^S,P^S\right) dP = -dD, \end{aligned}$$

$$\begin{aligned} dr_i &= \left(1-a^S\right)\pi^m dD_i = dr \equiv \left(1-a^S\right)\pi^m dD, \\ dr_j &= -\left(1-a^S\right)\pi^m dD = -dr. \end{aligned}$$

These expected changes in the revenues offered by the two platforms induce in turn a further adjustment in platforms' app bases, given by:

$$\begin{aligned} dy_i' &= \partial_1 Y\left(r^S,r^S\right) dr_i + \partial_2 Y\left(r^S,r^S\right) dr_j = dy' \equiv \left[\partial_1 Y\left(r^S,r^S\right) - \partial_2 Y\left(r^S,r^S\right)\right] dr, \\ dy_j' &= \partial_1 Y\left(r^S,r^S\right) dr_i + \partial_2 Y\left(r^S,r^S\right) dr_j = -\left[\partial_1 Y\left(r^S,r^S\right) - \partial_2 Y\left(r^S,r^S\right)\right] dr = -dy'. \end{aligned}$$

Combining the above observations leads to:

$$\begin{aligned} dy' &= \left[\partial_1 Y\left(r^S,r^S\right) - \partial_2 Y\left(r^S,r^S\right)\right] dr \\ &= \left[\partial_1 Y\left(r^S,r^S\right) - \partial_2 Y\left(r^S,r^S\right)\right]\left(1-a^S\right)\pi^m \\ &\quad \times \left[\partial_1 D\left(P^S,P^S\right) - \partial_2 D\left(P^S,P^S\right)\right]\left[\partial_1 P^e\left(\sigma^S,\sigma^S\right) - \partial_2 P^e\left(\sigma^S,\sigma^S\right)\right]\left(s^m + a^S\pi^m\right) dy \\ &= A^S dy. \end{aligned}$$

The stability of the continuation equilibrium requires $|dy'| < dy$, which in turn implies $A^S < 1$. ■

It follows from Claim that:

$$\partial_2 y^*\left(a^S,a^S\right) = \mathcal{D}^S + \mathcal{I}^S = \mathcal{D}^S + A^S\partial_2 y^*\left(a^S,a^S\right),$$

or:

$$\left(1-A^S\right)\partial_2 y^*\left(a^S,a^S\right) = \mathcal{D}^S.$$

Given Claim A.2, we can divide both sides of the above equation, which yields (17).

A.6 Proof of Corollary 2

From Proposition 2, $a^C = a^S$, characterized by the first-order condition

$$\partial_1 \sigma^* \left(a^S, a^S\right) + \partial_2 \sigma^* \left(a^S, a^S\right) = 0.$$

Furthermore, from Proposition 1 and Lemma 3 and $\mathcal{D}^S = 0$, with independent development we have:

$$\partial_2 \sigma^* \left(a^S, a^S\right) = \left(1 - a^S\right) \pi^m \partial_2 y^* \left(a^S, a^S\right) = 0.$$

It follows that a^S satisfies $\partial_1 \sigma^* \left(a^S, a^S\right) = 0$, and thus coincides with a^σ.

A.7 Proof of Proposition 3

We have:

$$\begin{aligned} a^C \gtreqless a^S \quad & \Longleftrightarrow \quad \partial_2 y^* \left(a^S, a^S\right) \lesseqgtr 0 \\ & \Longleftrightarrow \quad \mathcal{D}^S = -\pi^m D^S \partial_2 Y \left(r^S, r^S\right) \lesseqgtr 0 \\ & \Longleftrightarrow \quad \partial_2 Y \left(r^S, r^S\right) \gtreqless 0, \end{aligned}$$

where the first equivalence stems from Proposition 1, the second one from Lemma 3 and the third one from $\pi^m D^S > 0$. The following lemma completes the proof:

Lemma A.3 *We have:*

$$\partial_2 Y \left(r^S, r^S\right) \gtreqless 0 \quad \textit{if and only if} \quad s \gtreqless 0. \tag{A.15}$$

Proof. We consider in turn the cases of economies and diseconomies of scope.

• Case 1: $s > 0$. Consider a marginal increase in r_j, starting from $r_i = r_j = r^S$. We then have:

$$r_i + r_j - k > (r_i - k_i) + (r_j - k_j). \tag{A.16}$$

Developers at the margin between developing or not their apps for $\mathcal{P}_i$ are therefore as follows (see Figure 3):

- First of all, no such developer can be indifferent between developing for $\mathcal{P}_i$ only and developing for $\mathcal{P}_j$ only. This would require $r_i - k_i = r_j - k_j > 0$, which,

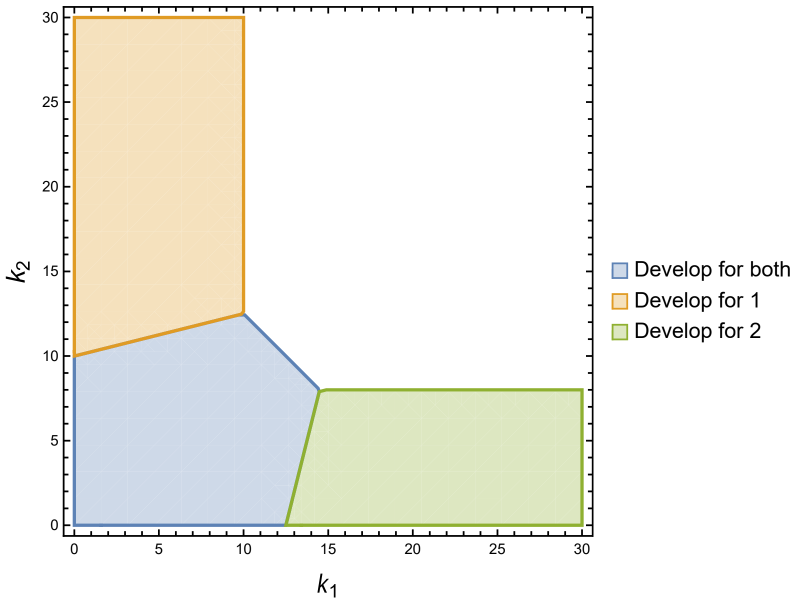

Figure 3: Economies of scope ($s > 0$) [Illustration for $r_1 = 10$ and $r_2 = 6$ and $\kappa(k_1, k_2; s) = \exp(-s)(k_1 + k_2)$, with $s = 2/10$].

together with ([A.16]), would imply $r_i + r_j - k > 0$. The developer would therefore *strictly* prefer developing its app for both platforms; hence, it could not be marginal. In the same vein, we do not need to consider those indifferent between developing for both platforms and developing for $\mathcal{P}_i$ only (and strictly preferring these options to not developing for $\mathcal{P}_i$), as they are not marginal for $\mathcal{P}_i$.

- Concerning those indifferent between developing for $\mathcal{P}_i$ only (yielding $r_i - k_i$) and not developing at all (yielding 0) (for instance, those with k_j high enough), their decision does not depend on r_j (as r_j affects none of these options).
- Concerning those indifferent between developing for both platforms (yielding $r_i + r_j - k$) and developing for $\mathcal{P}_j$ only (yielding $r_j - k_j$) (for instance, those with k_j low enough), their decision again does not depend on r_j (as r_j affects both options in the same way).
- Finally, concerning those indifferent between developing their apps for both platforms (yielding $r_i + r_j - k$) or not developing at all (yielding 0); their decision does depend on r_j: increasing r_j encourages more developers to develop their apps for the two platforms.

It follows that $\partial_2 Y\left(r^S, r^S\right) > 0$.

• Case 2: $s < 0$. Consider again a marginal increase in r_j, starting from $r_i = r_j = r^S$. We then have:

$$r_i + r_j - k < (r_i - k_i) + (r_j - k_j). \quad (A.17)$$

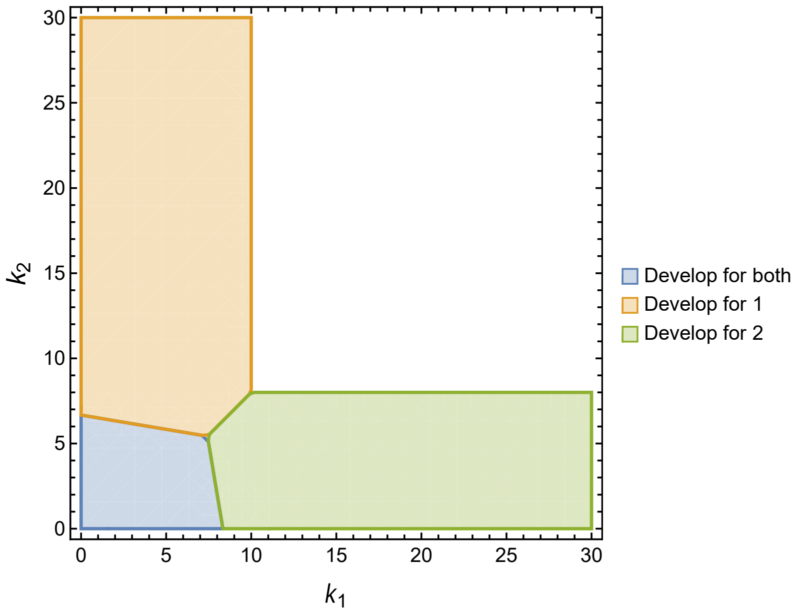

Figure 4: Diseconomies of scope ($s < 0$) [Illustration for $r_1 = 10$ and $r_2 = 6$ and $\kappa(k_1, k_2; s) = \exp(-s)(k_1 + k_2)$, where $s = -2/10$].

Developers at the margin between developing their apps for $\mathcal{P}_i$ and not developing are as follows (see Figure 4):

- First of all, no such developer can be indifferent between developing for both platforms (yielding $r_i + r_j - k$) and not developing at all. This would require $r_i + r_j - k = 0$, which, together with (A.17), would imply $\max\{r_i - k_i, r_j - k_j\} > 0$. The developer would therefore strictly prefer developing its app for at least one of the platforms; hence, it could not be marginal. Furthermore, as in case 1, we do not need to consider those indifferent between developing for both platforms and developing for $\mathcal{P}_i$ only (and strictly preferring these options to not developing for $\mathcal{P}_i$).

- As in case 1, regarding those indifferent between developing for $\mathcal{P}_i$ only and not developing at all and those indifferent between developing for both platforms and developing for $\mathcal{P}_j$ only, their decision does not depend on r_j.

- Finally, concerning those indifferent between developing their apps for $\mathcal{P}_i$ only (yielding $r_i - k_i$), and for $\mathcal{P}_j$ only (yielding $r_j - k_j$), their decision does depend on r_j: increasing r_j encourages those marginal developers to switch from $\mathcal{P}_i$ to $\mathcal{P}_j$.

It follows that $\partial_2 Y(\cdot) < 0$.

Summing-up, $s > 0$ implies $\partial_2 Y\left(r^S, r^S\right) > 0$, whereas $s < 0$ implies $\partial_2 Y\left(r^S, r^S\right) < 0$. Together, these two implications yield (A.15). ■

B Illustration

B.1 Proof of Lemma 4

The profit $\Pi_i = (P_i + \sigma_i) D(P_i, P_j)$ is strictly quasi-concave in P_i and maximal for

$$P_i = R(P_j) \equiv \frac{t - \sigma_i + P_j}{2}.$$

As this best-response has a slope lower than 1, the usual tâtonnement process converges towards a unique, stable equilibrium, in which each $\mathcal{P}_i$ charges $P_i = P^H(\sigma_i, \sigma_j)$, leading to

$$P_i + \sigma_i = 2tD(P_i, P_j) = t + \frac{\sigma_i - \sigma_j}{3},$$

and, thus, to $\Pi_i = \Pi^H(\sigma_i, \sigma_j)$.

B.2 Proof of Lemma 5

As mentioned in the text, from Lemma 2 and $\hat{D}'(a) = 0$, $a^W = -s^m/\pi^m$, implying $\hat{\sigma}(a^W) = 0$, regardless of α and t. Furthermore, from (19) – (22), $\hat{\sigma}(a)$ is independent of t, implying that a^S is also independent from it. This establishes part (ii) of the Lemma.

From Corollary 1, a^S generates a positive subsidy; hence:

$$s^m + a^S\pi^m > 0 \quad \text{and} \quad \hat{y}\left(a^S\right) > 0.$$

This, in turn, implies that a^S lies strictly between $a^W = -s^m/\pi^m$ and 1, which establishes part (i).

B.3 Proof of Proposition 4

We first establish uniqueness and existence for t large enough (part 1), before studying the properties of symmetric equilibria (part 2).

Part 1. As noted in the text, in stage 1a each $\mathcal{P}_i$ seeks to maximize its subsidy advantage, $\Delta^*(a_i, a_j)$. Furthermore, as $t \to +\infty$, the continuation equilibrium conditions (24) to (26) yield, up to $O(1/t)$:

$$D^*(a_i, a_j) \simeq \frac{1}{2},$$

and:

$$y_I^*(a_i, a_j) \simeq \hat{y}_I^*(a_i) \equiv (1-\alpha)F_I((1-a_i)\frac{\pi^m}{2}),$$
$$y_J^*(a_i, a_j) \simeq \hat{y}_J^*(a_1, a_2) \equiv \alpha F_J((1-\frac{a_1+a_2}{2})\pi^m).$$

Plugging-in these expressions in (28) yields, up to $O(1/t)$:

$$y^*(a_i, a_j) \simeq \hat{y}^*(a_i, a_j) \equiv \alpha \hat{y}_J^*(a_1, a_2) + (1-\alpha)\hat{y}_I^*(a_i),$$
$$\sigma^*(a_i, a_j) \simeq \hat{\sigma}^*(a_i, a_j) \equiv (s^m + a_i \pi^m)\hat{y}^*(a_i, a_j),$$
$$\Delta^*(a_i, a_j) \simeq \hat{\Delta}^*(a_i, a_j) \equiv \hat{\sigma}^*(a_i, a_j) - \hat{\sigma}^*(a_j, a_i).$$

In what follows, we consider the limit game Γ^∞ in which each platform seeks to maximize $\hat{\Delta}^*(a_i, a_j)$. We show that there exists a unique equilibrium, in which platforms' best-responses are moreover uniquely defined. By continuity, this establishes existence and uniqueness of the competitive equilibrium for t large enough.

We first note that we can restrict attention to non-negative subsidies:

Claim B.1 (non-negative subsidies) *In any equilibrium of game Γ^∞, both commissions are strictly higher than a^W.*

Proof. Consider a candidate equilibrium of game Γ^∞ yielding commissions a_1 and a_2, app bases $y \equiv \alpha \hat{y}_J^*(a_1, a_2)$ and $\{y_i \equiv (1-\alpha)\hat{y}_I^*(a_i)\}_{i=1,2}$, and subsidies $\{\sigma_i \equiv (s^m + a_i\pi^m)(y + y_i)\}_{i=1,2}$. Without loss of generality, suppose that $a_i \leq a_j$, implying $y_i \geq y_j$, and let $\Delta_i \equiv \sigma_i - \sigma_j$ denote $\mathcal{P}_i$'s subsidy advantage; finally, following a deviation to $a_i' \neq a_i$, let $y' \equiv \alpha \hat{y}_J^*(a_i', a_j)$, $y_i' \equiv (1-\alpha)\hat{y}_I^*(a_i')$, $\sigma_i' \equiv (s^m + a_i\pi^m)(y' + y_i')$, $\sigma_j' \equiv (s^m + a_j\pi^m)(y' + y_j)$ and $\Delta_i' \equiv \sigma_i' - \sigma_j'$ denote the resulting app bases, subsidies and subsidy advantage. We have:

- If $a_i < a^W$, then $\sigma_i < 0$ (as $s^m + a_i\pi^m < 0$ and $y + y_i \geq \alpha \hat{y}_J^*(a^W, 1) + (1-\alpha)\hat{y}_I^*(a^W) > 0$); consider now a deviation to $a_i' = a^W$, implying $\sigma_i' = 0$ (as $s^m + a_i'\pi^m = 0$), $y' \leq y$ and $y_i' \leq y_i$. We have:
  - if $a_j < a^W$, then $\sigma_i \leq \sigma_j$ (as $s^m + a_i\pi^m \leq s^m + a_j\pi^m < 0$ and $y_i \geq y_j$) and $\sigma_j' < 0 = \sigma_i'$; hence, $\Delta_i \leq 0 < -\sigma_j' = \Delta_i'$, implying that the deviation is strictly profitable;
  - if instead $a_j = a^W$, then $\sigma_j = \sigma_j' = 0$; hence, $\Delta_i = \sigma_i - 0 < 0 = \Delta_i'$, implying that the deviation is again strictly profitable;

- finally, if $a_j > a^W$, then $\sigma_j \geq \sigma'_j \geq 0$ (as $s^m + a_j\pi^m > 0$ and $y \geq y' \geq 0$); hence, $\Delta_i = \sigma_i - \sigma_j < \sigma'_i - \sigma'_j = \Delta'_i$ (as $\sigma_i < 0 = \sigma'_i$ and $\sigma_j \geq \sigma'_j$), implying that the deviation is once more strictly profitable.

- If instead $a_i = a^W < a_j$, then $\sigma_i = 0$ (as $s^m + a_i\pi^m = 0$); two cases can be distinguished:
  - if $\sigma_j > 0$, implying $\Delta_i = -\sigma_j < 0$, a deviation to $a'_i = a_j$ would lead to $\sigma'_i = \sigma'_j$ and $\Delta'_i = 0 > \Delta_i$, and would therefore be strictly profitable.
  - if instead $\sigma_j = 0$ (which can happen if $a_j = 1$ and $\alpha = 0$, implying $y = y_j = 0$), then $\Delta_j = \Delta_i = 0$; a deviation by j to, say, $a'_j = 0$ would lead to $\sigma'_i = 0$ (as $s^m + a_i\pi^m = 0$) and $\Delta'_j = \sigma'_j = s^m \hat{y}^*_I(0) > 0 = \Delta_j$, and would therefore be strictly profitable.

- Finally, if $a_i = a_j = a^W$, then $\sigma_i = \sigma_j = \Delta_i = 0$ (as $s^m + a_i\pi^m = s^m + a_j\pi^m = 0$); a deviation to, say $a'_i = 0$ would lead to $\sigma'_j = 0$ (as $s^m + a_j\pi^m = 0$) and $\Delta'_i = \sigma'_i = s^m(y' + y'_i) = s^m\left[\alpha\hat{y}^*_J\left(a^W, 0\right) + (1-\alpha)\hat{y}^*_I(0)\right] > 0 = \Delta_i$, and would therefore be strictly profitable.

It follows that, in any equilibrium of game Γ^∞, the commissions a_1 and a_2 are strictly higher than a^W. ■

Any commission $a > 1$ discourages app development and is formally equivalent to $a' = 1$. Hence, it follows from Claim B.1 that, without loss of generality we can restrict attention to commissions lying in the range $(a^W, 1]$. We have:

$$\begin{aligned}
\frac{\partial\hat{\Delta}^*(a_i, a_j)}{\partial a_i} &= \pi^m\left[(1-\alpha)F_I\left(\frac{(1-a_i)\pi^m}{2}\right) + \alpha F_J\left(\left(1 - \frac{a_1+a_2}{2}\right)\pi^m\right)\right] \\
&\quad -(s^m + a_i\pi^m)(1-\alpha)f_I\left(\frac{(1-a_i)\pi^m}{2}\right)\frac{\pi^m}{2} \\
&\quad -\pi^m(a_i - a_j)\alpha f_J\left(\left(1 - \frac{a_1+a_2}{2}\right)\pi^m\right)\frac{\pi^m}{2}.
\end{aligned}$$

Next, we show that any equilibrium must be symmetric:

Claim B.2 (symmetry) *In any equilibrium of game Γ^∞, commissions are symmetric.*

Proof. Consider an asymmetric candidate equilibrium and, without loss of generality, suppose that $(a^W <)a_i < a_j(\leq 1)$; we then have:

$$\delta(a_i, a_j) \equiv \frac{\partial\hat{\Delta}^*(a_i, a_j)}{\partial a_i} - \frac{\partial\hat{\Delta}^*(a_j, a_i)}{\partial a_j} = \alpha\delta_J(a_i, a_j) + (1-\alpha)\delta_I(a_i, a_j),$$

where

$$\delta_J(a_i, a_j) \equiv (a_j - a_i) f_J\left(\left(1 - \frac{a_1 + a_2}{2}\right)\pi^m\right)(\pi^m)^2 > 0,$$

where the inequality stems from $a_j > a_i$ and $f_J((1-(a_1+a_2)/2)\pi^m) \geq f_J((1-a_i)\pi^m) > 0$, and

$$\begin{aligned}\delta_I(a_i, a_j) &\equiv \left[F_I\left(\frac{(1-a_i)\pi^m}{2}\right) - F_I\left(\frac{(1-a_j)\pi^m}{2}\right)\right]\pi^m \\ &\quad - \left[(s^m + a_i\pi^m) f_I\left(\frac{(1-a_i)\pi^m}{2}\right) - (s^m + a_j\pi^m) f_I\left(\frac{(1-a_j)\pi^m}{2}\right)\right]\frac{\pi^m}{2} \\ &> 0,\end{aligned}$$

where the inequality stems from $F_I((1-a)\pi^m/2)$ being decreasing in a, whereas $s^m + a\pi^m$ and $f_I((1-a)\pi^m/2)$ are both non-negative and increasing in a, from Assumption 3. It follows that $\delta(a_i, a_j) > 0$ for any $\alpha \in [0,1]$ and any (a_i, a_j) such that $a^W < a_i < a_j \leq 1$. This, in turn, implies that the commissions (a_i, a_j) cannot constitute an equilibrium: if $a_j < 1$, we should have

$$\frac{\partial \hat{\Delta}^*(a_i, a_j)}{\partial a_i} = \frac{\partial \hat{\Delta}^*(a_j, a_i)}{\partial a_j} = 0,$$

implying $\delta(a_i, a_j) = 0$, a contradiction. If instead $a_j = 1$, we should have

$$\frac{\partial \hat{\Delta}^*(a_i, a_j)}{\partial a_i} = 0 \leq \frac{\partial \hat{\Delta}^*(a_j, a_i)}{\partial a_j},$$

implying $\delta(a_i, a_j) \leq 0$, another contradiction. ■

Building on this, we now establish uniqueness:

Claim B.3 (uniqueness) *If there exists an equilibrium of game Γ^∞, it is unique.*

Proof. For symmetric commissions, the derivative simplifies to:

$$\begin{aligned}\left.\frac{\partial \hat{\Delta}^*(a_i, a_j)}{\partial a_i}\right|_{a_1=a_2=a} &= \pi^m(1-\alpha)F_I\left(\frac{(1-a)\pi^m}{2}\right) + \pi^m \alpha F_J((1-a)\pi^m) \\ &\quad - (s^m + a\pi^m)(1-\alpha) f_I\left(\frac{(1-a)\pi^m}{2}\right)\frac{\pi^m}{2}. \qquad (B.1)\end{aligned}$$

For $\alpha = 1$, it boils down to $F_J((1-a)\pi^m)\pi^m \geq 0$, implying $a^C(\alpha) = 1$. For $\alpha < 1$, we have:

$$\left.\frac{\partial \hat{\Delta}^*(a_i, a_j)}{\partial a_i}\right|_{a_1=a_2=1} = -(s^m + \pi^m)(1-\alpha) f_I(0)\frac{\pi^m}{2} < 0.$$

It follows that any symmetric equilibrium satisfies the first-order condition $\left.\frac{\partial\hat{\Delta}^*(a_i,a_j)}{\partial a_i}\right|_{a_1=a_2=a} = 0$, which can be expressed as $\phi_C(a;\alpha)=0$, where (dividing by $(1-\alpha)\pi^m$):

$$\phi_C(a;\alpha) \equiv F_I\left(\frac{(1-a)\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}F_J\left((1-a)\pi^m\right) - \frac{s^m + a\pi^m}{2} f_I\left(\frac{(1-a)\pi^m}{2}\right).$$

$\phi_C(a;\alpha)$ is strictly decreasing in a under Assumption 3, and it satisfies (using $s^m + a^W\pi^m = 0$)

$$\phi_C\left(a^W;\alpha\right) = F_I\left(\frac{s^m+\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}F_J\left(s^m+\pi^m\right) > 0$$

and (using $F_I(0) = F_J(0) = 0$)

$$\phi_C(1;\alpha) = -\frac{s^m+\pi^m}{2} f_I(0) < 0,$$

where the inequalities stem from $f_I(0) > 0$ (from Assumption 3). It follows that there is a unique candidate symmetric equilibrium, which is moreover such that $a^C(\alpha) \in \left(a^W, 1\right)$ for $\alpha < 1$, and $a^C(\alpha) = 1$ for $\alpha = 1$. ■

To establish existence, we show that the function

$$\psi(a) \equiv \hat{\Delta}^*\left(a, a^C(\alpha)\right)$$

is indeed maximal for $a = a^C(\alpha)$. We first note that:

$$\begin{aligned}
\psi'(a) \quad = \quad & \pi^m(1-\alpha)F_I\left(\frac{(1-a)\pi^m}{2}\right) \\
& -\left(s^m + a\pi^m\right)(1-\alpha)f_I\left(\frac{(1-a)\pi^m}{2}\right)\frac{\pi^m}{2} \\
& +\pi^m\alpha F_J\left(\frac{(1-a)\pi^m}{2} + \frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right) \\
& -\pi^m\left(a - a^C(\alpha)\right)\alpha f_J\left(\frac{(1-a)\pi^m}{2} + \frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right)\frac{\pi^m}{2},
\end{aligned}$$

which by construction satisfies $\psi'\left(a^C(\alpha)\right)=0$, and

$$\begin{aligned}\psi''(a) &\simeq -2\pi^m(1-\alpha)f_I\left(\frac{(1-a)\pi^m}{2}\right)\frac{\pi^m}{2}\\ &+\left(s^m+a\pi^m\right)(1-\alpha)f_I'\left(\frac{(1-a)\pi^m}{2}\right)\left(\frac{\pi^m}{2}\right)^2\\ &-2\pi^m\alpha f_J\left(\frac{(1-a)\pi^m}{2}+\frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right)\frac{\pi^m}{2}\\ &+\pi^m\left(a-a^C(\alpha)\right)\alpha f_J'\left(\frac{(1-a)\pi^m}{2}+\frac{\left(1-a^C(\alpha)\right)\pi^m}{2}\right)\left(\frac{\pi^m}{2}\right)^2.\end{aligned}$$

For $a\geq a^C(\alpha)$, the second-order derivative is negative, as $f_I(\cdot)>0\geq f_I'(\cdot)$ and $f_J(\cdot)>0\geq f_J'(\cdot)$ under Assumption 3. Hence, in the range $a\geq a^C(\alpha)$, $\psi(a)$ is maximal for $a=a^C(\alpha)$.

Furthermore, for $a\leq a^C(\alpha)$, we have $\psi(a)\leq\hat{\psi}(a)$, where:

$$\begin{aligned}\psi(a)\leq\hat{\psi}(a)\equiv\quad &\left(s^m+a\pi^m\right)(1-\alpha)F_I\left(\frac{(1-a)\pi^m}{2}\right)\\ &-\left[s^m+a^C(\alpha)\pi^m\right](1-\alpha)F_I\left(\frac{\left[1-a^C(\alpha)\right]\pi^m}{2}\right)\\ &+\pi^m\left[a-a^C(\alpha)\right]\alpha F_J\left(\left[1-a^C(\alpha)\right]\pi^m\right)\end{aligned}$$

satisfies

$$\begin{aligned}\hat{\psi}'(a) &= \pi^m(1-\alpha)F_I\left(\frac{(1-a)\pi^m}{2}\right)-\left(s^m+a\pi^m\right)(1-\alpha)f_I\left(\frac{(1-a)\pi^m}{2}\right)\frac{\pi^m}{2}\\ &+\pi^m\alpha F_J\left(\left[1-a^C(\alpha)\right]\pi^m\right),\end{aligned}$$

By construction, $\hat{\psi}\left(a^C(\alpha)\right)=\psi\left(a^C(\alpha)\right)=0$ and $\hat{\psi}'\left(a^C(\alpha)\right)=\psi'\left(a^C(\alpha)\right)=0$. Furthermore:

$$\begin{aligned}\hat{\psi}''(a) &= -\pi^m(1-\alpha)f_I\left(\frac{(1-a)\pi^m}{2}\right)\pi^m\\ &+\left(s^m+a\pi^m\right)(1-\alpha)f_I'\left(\frac{(1-a)\pi^m}{2}\right)\left(\frac{\pi^m}{2}\right)^2\\ &< 0,\end{aligned}$$

where the inequality stems from $f_I(\cdot)>0\geq f_I'(\cdot)$ under Assumption 3. It follows that, in the range $a\leq a^C(\alpha)$, $\psi(a)$ is again maximal for $a=a^C(\alpha)$.

Part 2. We now focus on symmetric candidate equilibria, in which both platforms thus set the same commission a. Suppose that $\mathcal{P}_1$, say, deviates to some $a_1\neq a$, and let $y_{Ii}(a_1) \equiv y_I^*(a_i, a_j)$ (with the convention $a_2 = a$), $y_J(a_1) \equiv y_J^*(a_i, a_j)$, and $y_i(a_1) \equiv y^*(a_i, a_j)$ denote the resulting app bases, and $D_i(a_1) \equiv D^*(a_i, a_j)$ and $\Delta_i(a_1) \equiv \Delta^*(a_i, a_j)$ denote the user base and the subsidy advantage of $\mathcal{P}_i$ (by construction, $D_1 + D_2 = 1$); finally, let $\hat{y}_I$, $\hat{y}_J$, $\hat{y}$, $\hat{D}$ and $\hat{\Delta}$ denote the equilibrium values (by construction, $\hat{D} = 1/2$ and $\hat{\Delta} = 0$).

From (26) and (25), we have (using $a_2 = a$ and $D_1 + D_2 = 1$):

$$\begin{aligned} y_{I1} &= F_I\left((1-a_1)\,\pi^m D_1\right), \\ y_{I2} &= F_I\left((1-a)\,\pi^m (1-D_1)\right), \end{aligned}$$

leading to:

$$\frac{dy_{I1}}{da_1} = f_I\left((1-a_1)\pi^m D_1\right)\pi^m[(1-a_1)\frac{dD_1}{da_1} - D_1], \tag{B.2}$$

$$\frac{dy_{I2}}{da_1} = -f_I\left((1-a)\pi^m(1-D_1)\right)\pi^m(1-a)\frac{dD_1}{da_1}. \tag{B.3}$$

Likewise, from (27) and (25), we have (using again $a_2 = a$ and $D_1 + D_2 = 1$):

$$y_J = F_J\left((1-a)\,\pi^m + (a-a_1)\,\pi^m D_1\right), \tag{B.4}$$

leading to:

$$\frac{dy_J}{da_1} = f_J\left((1-a)\,\pi^m + (a-a_1)\,\pi^m D_1\right)\pi^m[(a-a_1)\frac{dD_1}{da_1} - D_1]. \tag{B.5}$$

In addition, from (28), we have:

$$\Delta_1 = s^m(1-\alpha)(y_{I1} - y_{I2}) + \pi^m(1-\alpha)(a_1 y_{I1} - a y_{I2}) + \pi^m\,(a_1 - a)\,\alpha y_J.$$

Differentiating leads to (using $y_{I1} + y_J = y_1$):

$$\begin{aligned} \frac{d\Delta_1}{da_1} &= s^m(1-\alpha)(\frac{dy_{I1}}{da_1} - \frac{dy_{I2}}{da_1}) + \pi^m(1-\alpha)(a_1\frac{dy_{I1}}{da_1} - a\frac{dy_{I2}}{da_1}) \\ &\quad + \pi^m\alpha\,(a_1 - a)\,\frac{dy_J}{da_1} + \pi^m(1-\alpha)y_{I1} + \pi^m\alpha y_J \\ &= (s^m + a_1\pi^m)(1-\alpha)\frac{dy_{I1}}{da_1} - (s^m + a\pi^m)(1-\alpha)\frac{dy_{I2}}{da_1} \\ &\quad + (a_1 - a)\pi^m\alpha\frac{dy_J}{da_1} + \pi^m y_1. \end{aligned} \tag{B.6}$$

Finally, differentiating (24) yields:

$$\frac{dD_1}{da_1} = \frac{1}{6t}\frac{d\Delta_1}{da_1}. \tag{B.7}$$

In equilibrium, we must have $d\Delta_1/da_1 = 0$. It then follows from (B.3) and (B.7) that

$$\frac{dy_{I2}}{da_1} = \frac{dD_1}{da_1} = 0$$

and from (B.2) that (using $D_1 = \hat{D} = 1/2$ and $y_{I1} = \hat{y}_I$)

$$\frac{dy_{I1}}{da_1} = -\frac{1}{2} f_I\left(\frac{(1-a_1)\pi^m}{2}\right)\pi^m.$$

Using these observations and evaluating (B.6) at equilibrium, where $a_1 = a$, $D_1 = 1/2$ and $y_1 = \hat{y}$, yields:

$$0 = \left[\hat{y} - (s^m + a\pi^m)\frac{1-\alpha}{2} f_I\left(\frac{(1-a_1)\pi^m}{2}\right)\right]\pi^m,$$

which, using (26) and (27), amounts to (B.1). It then follows from the analysis of part 1 that there is a unique symmetric equilibrium, where $a = a^C(\alpha)$.

Furthermore, $\phi_C(a;\alpha)$ is strictly increasing in α, implying that $a^C(\alpha)$ is also strictly increasing in α. Finally, for $a^C(1) = 1 > a^S(1)$ and, for $\alpha < 1$, $a^S(\alpha)$ satisfies the first-order condition $\hat{\sigma}'(a) = 0$ which, re-arranging, can be expressed as $\phi_S\left(a^S;\alpha\right) = 0$, where:

$$\begin{aligned}\phi_S(a;\alpha) \equiv\ & \left[F_I\left(\frac{(1-a)\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}F_J\left((1-a)\pi^m\right)\right] \\ & - \left(s^m + a^S\pi^m\right)\left[\frac{1}{2}f_I\left(\frac{(1-a)\pi^m}{2}\right) + \frac{\alpha}{1-\alpha}f_J((1-a)\pi^m)\right].\end{aligned}$$

The first bracketed term is strictly decreasing in a whereas the second one is weakly increasing in a under Assumption 3; hence, $\phi_S(a;\alpha)$ is also strictly decreasing in a in the relevant range where $s^m + a\pi^m > 0$. In addition, we have:

$$\phi_C(a;\alpha) - \phi_S(a;\alpha) = (s^m + a\pi^m)\frac{\alpha}{1-\alpha}f_J((1-a)\pi^m) \geq 0,$$

where the inequality is strict for $\alpha > 0$. It follows that $a^C(0) = a^S(0)$ and $a^C(\alpha) > a^S(\alpha)$ for $\alpha > 0$.

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Online Appendix

O-A Alternative pricing strategies

We show here that the insights of Section 3 carry over when platforms charge wholesale prices (per consumer served) to app developers instead of ad valorem commissions, but no longer do so when platforms rely instead on fixed fees (independent of the number of consumers served).

O-A.1 Wholesale prices

We start with the case where platforms charge wholesale prices. The analysis is similar to that exposed in Section 3, and we only sketch here the main steps.

In stage 2, when facing a wholesale price w, a developer sets

$$p^m(w) \equiv \arg\max_p (p - w)\, d(p),$$

which generates a profit $\pi^m(w) \equiv (p^m(w) - w)\, d(p^m(w))$ for the developer, a revenue $\rho^m(w) \equiv wd(p^m(w))$ for the platform, and a surplus $s^m(w) \equiv s(p^m(w))$ for consumers. Thus, if $\mathcal{P}_i$ sets a wholesale price w_i, its profit is

$$\Pi_i = (p_i + \rho^m(w_i)\, y_i) D(p_i - s_i y_i, p_j - s_j y_j) = (P_i + \sigma_i)\, D(P_i, P_j),$$

where the subsidy is now equal to

$$\sigma_i = [s_i + \rho^m(w_i)]\, y_i.$$

Let $y^*(w_i, w_j)$ denote $\mathcal{P}_i$'s app base at the end of stage 1, as a function of the wholesale prices, and define:

$$\sigma^*(w_i, w_j) \equiv [s^m(w_i) + \rho^m(w_i)]\, y^*(w_i, w_j),$$

which satisfies

$$\partial_2 \sigma^*(w_i, w_j) = [s^m(w_i) + \rho^m(w_i)]\, \partial_2 y^*(w_i, w_j),$$

and

$$\begin{aligned}
P^*(w_i,w_j) &= P^e(\sigma^*(w_i,w_j),\sigma^*(w_j,w_i)),\\
D^*(w_i,w_j) &= D(P^*(w_i,w_j),P^*(w_j,w_i)),\\
\Pi^*(w_i,w_j) &= [P^*(w_i,w_j)+\sigma^*(w_i,w_j)]\,D^*(w_i,w_j),\\
r^*(w_i,w_j) &\equiv \pi^m(w_i)D^*(w_i,w_j).
\end{aligned}$$

O-A.1.1 Benchmarks

For $w_1 = w_2 = w$, consumer surplus and social welfare are now respectively given by:

$$\hat{S}(w) \equiv \int_{P^*(w,w)}^{+\infty} 2D(P,P)\,dP \text{ and } \hat{W}(w) \equiv \hat{S}(w) + \hat{\Pi}_D(w) + 2\Pi^*(w,w)$$

where

$$\hat{\Pi}_D(w) \equiv \int_{\mathbb{R}^3_+} \pi_D(r^*(w,w),\mathbf{k})\,d\bar{F}(\mathbf{k}).$$

We have:

Lemma O-A.1 (benchmarks – wholesale prices) *Maximizing consumer surplus, $\hat{S}(w)$, or platforms' profit, $\hat{\Pi}_P(w)$, amounts to maximizing platforms' subsidy, $\hat{\sigma}(w)$. Maximizing social welfare requires a strictly lower wholesale price. Formally:*

$$w^W \equiv \underset{w}{\operatorname{argmax}}\,\hat{W}(w) < w^S \equiv \underset{w}{\operatorname{argmax}}\,\hat{S}(w) = \underset{w}{\operatorname{argmax}}\,\sigma^*(w,w) = \underset{w}{\operatorname{argmax}}\,\Pi^*(w,w).$$

Proof. Maximizing consumer surplus amounts again to minimizing the quality-adjusted price, which from Assumption 1 (a) requires maximizing the subsidy; likewise, from Assumption 1 (b), maximizing the profits of the platforms amounts to maximizing the subsidy.

The individual and aggregate app profit, $\pi_D(r^*(w,w),\mathbf{k})$ and $\hat{\Pi}_D(w)$, both vary like $r^*(w,w)$. Furthermore, slightly reducing w from w^S strictly increases developers' per consumer profit, $\pi^m(w)$, and has only a second-order effect on the (subsidy and, thus, on the) consumer base $D^*(w,w)$. It follows that, starting from $w = w^S$, a slight reduction in w would increase $r^*(w,w)$ and, thus, the developers' profit. The rest of the proof follows the same steps as Lemma 2. ■

O-A.1.2 Platform competition

In what follows, we maintain the analogous of Assumption 2 for $\Pi^*(w_i,w_j)$. We have:

Proposition O-A.1 (platform competition – wholesale prices) *Platform competition yields higher (resp., lower) wholesale prices than those maximizing consumer surplus whenever raising one wholesale price reduces (resp., increases) the rival's app base. Formally:*

$$w^C \gtreqless w^S \quad \textit{if and only if} \quad \partial_2 y^* \left(w^S, w^S\right) \lesseqgtr 0.$$

Proof. By construction, w^S maximizes $\sigma^*(w, w)$. Hence

$$\partial_1 \sigma^*(w^S, w^S) + \partial_2 \sigma^*(w^S, w^S) = 0.$$

Therefore:

$$\begin{aligned}
\partial_1 \Pi^*(w^S, w^S) &= \partial_1 \Pi^e(\sigma^S, \sigma^S)\partial_1 \sigma^*(w^S, w^S) + \partial_2 \Pi^e(\sigma^S, \sigma^S)\partial_2 \sigma^*(w^S, w^S) \\
&= -\left[\partial_1 \Pi^e(\sigma^S, \sigma^S) - \partial_2 \Pi^e(\sigma^S, \sigma^S)\right] \partial_2 \sigma^*(w^S, w^S) \\
&= -\left[\partial_1 \Pi^e(\sigma^S, \sigma^S) - \partial_2 \Pi^e(\sigma^S, \sigma^S)\right] \left[s^m(w^S) + \rho^m(w^S)\right] \partial_2 y^*(w^S, w^S).
\end{aligned}$$

The remainder of the proof follows the same steps as the proof of Proposition 1. ■

The remainder of the analysis replicates that of Section 3.3. In particular, we still have

$$\partial_2 y^* \left(w^S, w^S\right) = \mathcal{D}^S + \mathcal{I}^S = \frac{\mathcal{D}^S}{1 - A^S}, \tag{O-A.1}$$

with the caveat that the direct impact of the rival's price on the revenue offered to developers, equal to $-\pi^m D^S$ in the case of commissions, becomes

$$\left.\frac{d\pi^m(w)}{dw}\right|_{w=w^S} D^S = -d^S D^S,$$

where $d^S \equiv d(p^m(w^S))$, and the indirect impact through the consumer bases, equal to $(1-a)\,\pi^m \partial_h D^*$ in the case of commissions, becomes

$$\pi^S \partial_h D^* \left(w^S, w^S\right),$$

where $\pi^S \equiv \pi^m\left(w^S\right)$. As a result, the direct and indirect effects are given by:

$$\begin{aligned}
\mathcal{D}^S &= -d^S D^S \partial_2 Y\left(r^S, r^S\right), \\
\mathcal{I}^S &= \pi^S \left[\partial_2 D^*\left(w^S, w^S\right) \partial_1 Y\left(r^S, r^S\right) + \partial_1 D^*\left(w^S, w^S\right) \partial_2 Y\left(r^S, r^S\right)\right],
\end{aligned} \tag{O-A.2}$$

and the factor A^S is now equal to

$$A^S \equiv \left[\partial_1 Y(\cdot) - \partial_2 Y(\cdot)\right]\left[\partial_1 D(\cdot) - \partial_2 D(\cdot)\right]\left[\partial_1 P^e(\cdot) - \partial_2 P^e(\cdot)\right] \pi^S \left(s^S + \rho^S\right),$$

where $s^S + \rho^S = s\left(p^m\left(w^S\right)\right) + \rho^m\left(w^S\right)$ denotes the marginal impact that an increase in app base has on a platform's subsidy, and the stability of the price-setting game still guarantees $A^S < 1$. Building on this yields:

Proposition O-A.2 (cost externalities – wholesale prices) *We have:*

(*i*) *In case of independent development decisions (i.e., when $k = k_1 + k_2$, with k_1 and k_2 i.i.d. according to $F(\cdot)$), platform competition yields the wholesale prices that maximize consumer surplus: $w^C = w^S$.*

(*ii*) *Under Assumption S: $w^C \gtrless w^S$ if and only if $s \gtrless 0$.*

Proof. Part (*i*) follows directly from Proposition O-A.1, (O-A.1), (O-A.2) and the observation that, in the case of independent development decisions, $Y\left(r_i, r_j\right) = F\left(r_i\right)$ and $\partial_2 Y\left(\cdot\right) = 0$.

Part (*ii*). We have:

$$\begin{aligned} w^C \gtrless w^S &\iff \partial_2 y^*\left(w^S, w^S\right) \lesseqgtr 0 \\ &\iff \mathcal{D}^S = -d^S D^S \partial_2 Y\left(r^S, r^S\right) \lesseqgtr 0 \\ &\iff \partial_2 Y\left(r^S, r^S\right) \gtrless 0, \end{aligned}$$

where the first equivalence stems from Proposition O-A.1, the second one from (O-A.1) and the third one from $d^S D^S > 0$. The conclusion follows from Lemma A.3. ■

O-A.2 Fixed fees

We now turn to the case where platforms charge fixed fees and show that the above insights no longer hold. The main difference with the previous analyses is that these fees no longer directly affect platforms' subsidies (they only affect them indirectly, through the impact on app bases), but still have a direct (separable) impact on platforms' profits. As a result, consumers' and platforms' interests no longer coincide, even in the case of independent development decisions.

In stage 2, when facing a fixed fee φ, a developer sets

$$p^m = \arg\max_p pd(p) - \varphi,$$

as in the case of commissions. This now generates a profit $\pi^m - \varphi$ for the developer, a revenue φ for the platform, and a surplus s^m for consumers. Thus, if $\mathcal{P}_i$ sets a fee φ_i, its profit is

$$\Pi_i = p_i D(p_i - s^m y_i, p_j - s^m y_j) + \varphi_i y_i = \left(P_i + \sigma_i\right) D(P_i, P_j) + \varphi_i y_i,$$

where the subsidy is now equal to

$$\sigma_i = s^m y_i.$$

At the beginning of stage 2, φ_i and y_i are both given. Hence, competition for consumers leads again to $P_i = P^e(\sigma_i, \sigma_j)$; however, the resulting profit is now equal to

$$\Pi_i = \Pi^e(\sigma_i, \sigma_j) + \varphi_i y_i.$$

Let $y^*(\varphi_i, \varphi_j)$ denote $\mathcal{P}_i$'s app base at the end of stage 1, as a function of the fees, and define:

$$\sigma^*(\varphi_i, \varphi_j) \equiv s^m y^*(\varphi_i, \varphi_j), \tag{O-A.3}$$

which satisfies

$$\partial_2 \sigma^*(\varphi_i, \varphi_j) = s^m \partial_2 y^*(\varphi_i, \varphi_j). \tag{O-A.4}$$

O-A.2.1 Consumers

As before, maximizing consumer surplus amounts to minimizing equilibrium prices, which in turn amounts to maximizing platforms' subsidies. Here, however, this boils down to maximizing developers' participation, which in turn amounts to maximizing their revenue, $\pi^m - \varphi$; hence, in the absence of any constraint on the fees, the surplus-maximizing fee would be $\varphi^S = -\infty$. Obviously, this cannot arise in equilibrium.

O-A.2.2 Competitive bottlenecks

As an alternative benchmark, following the competitive bottlenecks literature we now consider the sum of platforms' profits and consumer surplus, equal here to $2V(\varphi)$, where:

$$V(\varphi) \equiv \Pi^e(\sigma^*(\varphi, \varphi), \sigma^*(\varphi, \varphi)) + \varphi y^*(\varphi, \varphi) + \int_{P^e(\sigma^*(\varphi,\varphi), \sigma^*(\varphi,\varphi))}^{+\infty} D(P, P)\, dP.$$

Let $\varphi = \varphi^V$ denote the fee that maximizes this joint payoff, and $y^V \equiv y^*(\varphi^V, \varphi^V)$, $\sigma^V \equiv \sigma^*(\varphi^V, \varphi^V)$, $P^V \equiv P^e(\sigma^V, \sigma^V)$, $D^V \equiv D(P^V, P^V)$ denote the resulting app base, subsidy, consumer price and demand. The fee φ^V is characterized by the first-order condition:

$$\begin{aligned} 0 &= (\partial_1 \Pi^e + \partial_2 \Pi^e) \times (\partial_1 \sigma^* + \partial_2 \sigma^*) + \left[y^V + \varphi^V \times (\partial_1 y^* + \partial_2 y^*)\right] \\ &\quad - D^V \times (\partial_1 P^e + \partial_2 P^e) \times (\partial_1 \sigma^* + \partial_2 \sigma^*) \\ &= \left[\partial_1 \Pi^e + \partial_2 \Pi^e - D^V (\partial_1 P^e + \partial_2 P^e)\right] s^m (\partial_1 y^* + \partial_2 y^*) + \left[y^V + \varphi^V (\partial_1 y^* + \partial_2 y^*)\right] \\ &= \left\{\left[\partial_1 \Pi^e + \partial_2 \Pi^e - D^V (\partial_1 P^e + \partial_2 P^e)\right] s^m + \varphi^V\right\} (\partial_1 y^* + \partial_2 y^*) + y^V, \end{aligned} \tag{O-A.5}$$

where the second equality stems from (O-A.4) and, for $i = 1, 2$, $\partial_i \Pi^e$ and $\partial_i P^e$ are evaluated at $\sigma_1 = \sigma_2 = \sigma^V$, whereas $\partial_i y^*$ is evaluated at $\varphi_1 = \varphi_2 = \varphi^V$.

Using

$$\Pi^e(\sigma_i, \sigma_j) = \max_{P_i} \left\{(P_i + \sigma_i) D(P_i, P^e(\sigma_j, \sigma_i))\right\},$$

we have:

$$\begin{aligned} \partial_1 \Pi^e &= D^V + \left(P^V + \sigma^V\right) \partial_2 D \partial_2 P^e, \\ \partial_2 \Pi^e &= \left(P^V + \sigma^V\right) \partial_2 D \partial_1 P^e, \end{aligned}$$

where $\partial_2 D$ is evaluated at $P_1 = P_2 = P^V$. Furthermore, the first-order condition of the profit maximization with respect to P_i at stage 2 yields

$$D^V + \left(P^V + \sigma^V\right) \partial_1 D = 0.$$

Hence, (O-A.5) can be expressed as:

$$\begin{aligned} 0 &= \left\{\left[D^V + \left(P^V + \sigma^V\right) \partial_2 D (\partial_2 P^e + \partial_1 P^e) - D^V (\partial_1 P^e + \partial_2 P^e)\right] s^m + \varphi^V\right\} (\partial_1 y^* + \partial_2 y^*) + y^V \\ &= \left\{\left[D^V + \left(P^V + \sigma^V\right) (\partial_1 D + \partial_2 D) (\partial_1 P^e + \partial_2 P^e)\right] s^m + \varphi^V\right\} (\partial_1 y^* + \partial_2 y^*) + y^V. \end{aligned} \tag{O-A.6}$$

O-A.2.3 Platform competition

In a symmetric equilibrium, each $\mathcal{P}_i$ chooses $\varphi_i = \varphi^C$ to maximize:

$$\begin{aligned} \Pi_i &= \Pi^e(\sigma^*(\varphi_i, \varphi^C), \sigma^*(\varphi^C, \varphi_i)) + \varphi_i y^*(\varphi_i, \varphi^C) \\ &= \max_{P_i} \left[P_i + \sigma^*(\varphi_i, \varphi^C)\right] D(P_i, P^e(\sigma^*(\varphi^C, \varphi_i), \sigma^*(\varphi_i, \varphi^C))) + \varphi_i y^*(\varphi_i, \varphi^C). \end{aligned}$$

Let $y^C \equiv y^*\left(\varphi^C, \varphi^C\right)$, $\sigma^C \equiv \sigma^*\left(\varphi^C, \varphi^C\right)$, $P^C \equiv P^e\left(\sigma^C, \sigma^C\right)$, $D^C \equiv D\left(P^C, P^C\right)$ denote the resulting app base, subsidy, consumer price and demand. The fee φ^C is characterized by the first-order condition, which, using the envelope theorem, can be expressed as:

$$\begin{aligned} 0 &= \partial_1\sigma^* \times D^C + \left(P^C + \sigma^C\right) \times \partial_2 D \times \left(\partial_1 P^e \partial_2 \sigma^* + \partial_2 P^e \partial_1 \sigma^*\right) + \left(y^C + \varphi^C \times \partial_1 y^*\right) \\ &= D^C s^m \partial_1 y^* + \left(P^C + \sigma^C\right)\partial_2 D s^m \left(\partial_1 P^e \partial_2 y^* + \partial_2 P^e \partial_1 y^*\right) + y^C + \varphi^C \partial_1 y^* \qquad \text{(O-A.7)} \end{aligned}$$

where, for $i = 1, 2$, $\partial_i P^e$ is evaluated at $\sigma_1 = \sigma_2 = \sigma^C$, $\partial_i D$ is evaluated at $P_1 = P_2 = P^C$, whereas $\partial_i y^*$ is evaluated at $\varphi_1 = \varphi_2 = \varphi^C$.

At first glance, the first-order conditions (O-A.6) and (O-A.7) appear unlikely to coincide, suggesting that the insight obtained by Armstrong (2006) for the case of simultaneous competition on both sides, does not carry over to our sequential competition setting. To explore this further, we now study an illustrative example.

O-A.2.4 Illustration

As in Armstrong (2006), we focus on the case of independent development decisions and, for illustrative purposes, consider a Hotelling specification for consumer demand.

Competition for consumers As in Section 4, we consider the following consumer demand:

$$D\left(P_i, P_j\right) = \frac{1}{2} - \frac{P_i - P_j}{2t}.$$

From Lemma 4, in stage 2 we have $P_i = P^H\left(\sigma_i, \sigma_j\right)$ and $\Pi_i = \Pi^H\left(\sigma_i, \sigma_j\right) + \varphi_i y_i$, where:

$$P^H\left(\sigma_i, \sigma_j\right) \equiv t - \frac{2\sigma_i + \sigma_j}{3} \quad \text{and} \quad \Pi^H\left(\sigma_i, \sigma_j\right) \equiv \frac{t}{2}\left(1 + \frac{\sigma_i - \sigma_j}{3t}\right)^2.$$

In particular, in any symmetric equilibrium $\varphi_1 = \varphi_2 = \varphi^C$ with associated app base y^C and subsidy σ^C, the price, demand and profit are respectively:

$$P^C = t - \sigma^C, \quad D^C = \frac{1}{2}, \quad \text{and} \quad \Pi^C = \frac{t}{2} + \varphi^C y^C.$$

Competition for apps As in Section 3.3.2 on independent development decisions, we assume that developers face platform-specific costs k_1 and k_2 (together with $k = k_1 + k_2$), symmetrically and independently distributed across developers, with marginal c.d.f. $F(\cdot)$ and density $f(\cdot)$; we further assume that the (inverse reversed) hazard rate

$$h\left(k\right) = \frac{F\left(k\right)}{f\left(k\right)}$$

is weakly increasing.

In stage 1, in response to given fees (φ_1, φ_2) and expected consumer bases (D_1, D_2), developers' decisions lead to app bases (y_1, y_2) satisfying, for $i \neq j \in \{1, 2\}$:

$$y_i = F(\pi^m D_i - \varphi_i).$$

To go further, from now on we focus on the case where platforms are highly differentiated (i.e., $t \to +\infty$). We then have, up to $O(1/t)$:

$$D_i \simeq D^C = \frac{1}{2} \quad \text{and} \quad y_i \simeq F\left(\frac{\pi^m}{2} - \varphi_i\right),$$

implying:

$$\partial_1 y^* \simeq -f\left(\frac{\pi^m}{2} - \varphi_i\right) \quad \text{and} \quad \partial_2 y^* \simeq 0.$$

The condition (O-A.6), characterizing the fee that maximizes the joint payoff of the platforms and their consumers, thus boils down to (noting that $\partial_1 D + \partial_2 D = 0$):

$$\begin{aligned} 0 &= \left(D^V s^m + \varphi^V\right)\left(\partial_1 y^* + \partial_2 y^*\right) + y^V \\ &= \left(\frac{s^m}{2} + \varphi^V\right)\left[-f\left(\frac{\pi^m}{2} - \varphi^V\right) + 0\right] + F\left(\frac{\pi^m}{2} - \varphi^V\right) \\ &= f\left(\frac{\pi^m}{2} - \varphi^V\right)\left[h\left(\frac{\pi^m}{2} - \varphi^V\right) - \left(\frac{s^m}{2} + \varphi^V\right)\right]. \end{aligned}$$

The monotonicity of the hazard rate $h(\cdot)$ then ensures that $\varphi = \varphi^V$ is the unique solution to $\Phi^V(\varphi) = 0$, where

$$\Phi^V(\varphi) \equiv h\left(\frac{\pi^m}{2} - \varphi\right) - \left(\frac{s^m}{2} + \varphi\right)$$

is strictly decreasing in φ.

By contrast, the equilibrium condition (O-A.7) amounts to:

$$\begin{aligned} 0 &= \left\{\left[D^C + \left(P^C + \sigma^C\right)\partial_2 D \partial_2 P^e\right] s^m + \varphi^C\right\}\partial_1 y^* + \left(P^C + \sigma^C\right)\partial_2 D \partial_1 P^e s^m \partial_2 y^* + y^C \\ &= \left\{\left[\frac{1}{2} + t \times \frac{1}{2t} \times \left(\frac{-1}{3}\right)\right] s^m + \varphi^C\right\}\left(-f\left(\frac{\pi^m}{2} - \varphi^C\right)\right) + F\left(\frac{\pi^m}{2} - \varphi^C\right) \\ &= f\left(\frac{\pi^m}{2} - \varphi^C\right)\left[h\left(\frac{\pi^m}{2} - \varphi^C\right) - \left(\frac{s^m}{3} + \varphi^C\right)\right]. \end{aligned}$$

It follows that $\varphi = \varphi^C$ is the unique solution to $\Phi^C(\varphi) = 0$, where

$$\Phi^C(\varphi) \equiv h\left(\frac{\pi^m}{2} - \varphi\right) - \left(\frac{s^m}{3} + \varphi\right)$$

is also strictly decreasing in φ. Furthermore:

$$\Phi^C(\varphi) - \Phi^V(\varphi) = \frac{s^m}{6} > 0,$$

which, together with the monotonicity of $\Phi^C(\cdot)$ and $\Phi^V(\cdot)$, implies:

$$\varphi^C > \varphi^V.$$

For instance, when costs are uniformly distributed over $[0,1]$ (i.e., $F(k) = k$), we have:

$$\varphi^C = \frac{3\pi^m - 2s^m}{12} > \varphi^V = \frac{\pi^m - s^m}{4}.$$

Hence, the result obtained by Armstrong (2006) for the case of (independent development decisions, fixed fees and) simultaneous competition on both sides, does not hold in our sequential competition setting.

O-B Extensions

O-B.1 Multiple platforms

O-B.1.1 Proof of Lemma 6

Platforms' profits are strictly concave in their own quality-adjusted prices; the equilibrium is therefore characterized by the first-order conditions, given by, for $i = 1, \ldots, n$:

$$0 = \frac{\partial \bar{\Pi}_n^S\left(P_i, \bar{P}_i; \sigma_i\right)}{\partial P_i} = \bar{D}_n^S\left(P_i, \bar{P}_i\right) + (P_i + \sigma_i)\frac{\partial \bar{D}_n^S\left(P_i, \bar{P}_i\right)}{\partial P_i} = \frac{1}{n} - \frac{P_i - \bar{P}_i}{nt} - \frac{P_i + \sigma_i}{nt},$$

or:

$$0 = \frac{t}{n} - \frac{(2n-1)P_i - \sum_j P_j}{n(n-1)} - \frac{\sigma_i}{n}. \tag{O-B.1}$$

Summing these conditions for $i = 1, \ldots, n$ and re-arranging yields:

$$\sum_j P_j = nt - \sum_j \sigma_j.$$

Combining this with (O-B.1) leads to:

$$P_i = t - \frac{n\sigma_i + (n-1)\bar{\sigma}_i}{2n-1} = P_n^S(\sigma_i, \bar{\sigma}_i),$$

and, using $\Delta_i = \sigma_i - \bar{\sigma}_i$:

$$\begin{aligned} D_i &= \frac{1}{n} - \frac{P_i - \bar{P}_i}{nt} = \frac{1}{nt}\left(t + \frac{n-1}{2n-1}\Delta_i\right) = D_n^S(\Delta_i) \\ \Pi_i &= (P_i + \sigma_i) D_i = \frac{1}{nt}\left(t + \frac{n-1}{2n-1}\Delta_i\right)^2 = \Pi_n^S(\Delta_i). \end{aligned}$$

O-B.1.2 Proof of Lemma 7

Consumer surplus and social welfare are respectively given by:

$$\begin{aligned} \hat{S}_n^S(a) &\equiv u_0 - \frac{t}{4} - \hat{P}_n^S(a) = u_0 - \frac{5t}{4} + \hat{\sigma}_n^S(a), \\ \hat{W}_n^S(a) &\equiv \hat{S}_n^S(a) + n\hat{\Pi}_n^S(a) + \hat{\Pi}_D^S(a,n) = u_0 - \frac{t}{4} + (s^m + \pi^m)\hat{y}_n^S(a) - \hat{K}_n^S(a). \end{aligned}$$

We thus have:

$$\begin{aligned} \frac{d\hat{W}_n^S}{da}(a) &= (s^m + \pi^m)\frac{d\hat{y}_n^S}{da} - \frac{d\hat{K}_n^S}{da}(a) \\ &= (s^m + \pi^m)\left[-\alpha f_J((1-a)\pi^m)\pi^m - (1-\alpha) f_I\left((1-a)\frac{\pi^m}{n}\right)\frac{\pi^m}{n}\right] \\ &\quad + \alpha(1-a)\pi^m f_J((1-a)\pi^m)\pi^m + (1-\alpha) n \times (1-a)\frac{\pi^m}{n} f_I((1-a)\frac{\pi^m}{n})\frac{\pi^m}{n} \\ &= -(s^m + a\pi^m)\left[\alpha f_J((1-a)\pi^m)\pi^m + (1-\alpha) f_I((1-a)\frac{\pi^m}{n})\frac{\pi^m}{n}\right], \end{aligned}$$

and:

$$\begin{aligned} \frac{d^2\hat{W}_n^S}{da^2}(a) &= -\pi^m\left[\alpha f_J((1-a)\pi^m)\pi^m + (1-\alpha) f_I((1-a)\frac{\pi^m}{n})\frac{\pi^m}{n}\right] \\ &\quad + (s^m + a\pi^m)\left[\alpha f_J'((1-a)\pi^m)(\pi^m)^2 + (1-\alpha) f_I'((1-a)\frac{\pi^m}{n})\left(\frac{\pi^m}{n}\right)^2\right] \\ &< 0, \end{aligned}$$

where the inequality stems from Assumption 3. It follows that the welfare-maximizing commission is uniquely characterized by the first-order condition, leading to $a^W = -s^m/\pi^m$, regardless of n, α and t.

Maximizing consumer surplus requires maximizing platforms' subsidy, $\hat{\sigma}_n^S(a) = (s^m + a\pi^m)\hat{y}_n^S(a)$, where:

$$\hat{y}_n^S(a) = \alpha F_J((1-a)\pi^m) + (1-\alpha) F_I\left(\frac{(1-a)\pi^m}{n}\right). \quad \text{(O-B.2)}$$

It follows that $\hat{y}_n^S(a)$ and $\hat{\sigma}_n^S(a)$ are both independent of t; hence, the commission that maximizes consumer surplus is also independent of t. Furthermore, as the optimal value, $a_n^S(\alpha)$, maximizes $\hat{\sigma}_n^S(a)$, we have:

$$\hat{\sigma}_n^S\left(a_n^S(\alpha)\right) \geq \hat{\sigma}_n^S(0) = s^m\left[\alpha F_J(\pi^m) + (1-\alpha)F_I\left(\frac{\pi^m}{n}\right)\right] > 0.$$

It follows that $s^m + a_n^S(\alpha)\pi^m > 0$ and $\hat{y}_n^S\left(a_n^S(\alpha)\right) > 0$, implying that $a_n^S(\alpha)$ lies strictly between a^W and 1.

If $\alpha = 1$, then

$$\hat{\sigma}_n^S(a) = \hat{\sigma}_J(a) \equiv (s^m + a\pi^m)\hat{y}_J(a) = (s^m + a\pi^m)F_J((1-a)\pi^m),$$

which, in the relevant range where $s^m + a\pi^m > 0$, satisfies:

$$\begin{aligned}
\hat{\sigma}_J'(a) &= \pi^m\hat{y}_J(a) + (s^m + a\pi^m)\hat{y}_J'(a) = \pi^m\left[F_J((1-a)\pi^m) - (s^m + a\pi^m)f_J((1-a)\pi^m)\right], \\
\hat{\sigma}_J''(a) &= -(\pi^m)^2\left[2f_J((1-a)\pi^m) - (s^m + a\pi^m)f_J'((1-a)\pi^m)\right] < 0,
\end{aligned}$$

where the inequality stems from Assumption 3. Hence, regardless of n, $a_n^S(1) = a_J^S$, where $a_J^S \in \left(a^W, 1\right)$ is the unique solution in a to:

$$s^m + a\pi^m = \frac{F_J((1-a)\pi^m)}{f_J((1-a)\pi^m)}.$$

For $\alpha < 1$, as n goes to infinity, independent development tends to disappear, and so in the limit the subsidy is given by $\alpha\hat{\sigma}_J(a)$. It follows that, as n goes to infinity, $a_n^S(\alpha)$ tends to $a_J^S\left(\in\left(a^W, 1\right)\right)$.

Example 2 (uniform distribution) *In the particular case where development costs are uniformly distributed over* $[0,1]$ *(i.e.,* $F_J(k) = F_I(k) = k$*), we have:*

$$\hat{y}_n^S(a) = \alpha(1-a)\pi^m + (1-\alpha)(1-a)\frac{\pi^m}{n} = (1-a)\left[1 + (n-1)\alpha\right]\frac{\pi^m}{n},$$

and:

$$\frac{d\hat{\sigma}_n^S}{da}(a) = \pi^m\hat{y}_n^S(a) + (s^m + a\pi^m)\frac{d\hat{y}_n^S}{da}(a) = \left[1 + (n-1)\alpha\right]\frac{\pi^m}{n}(\pi^m - s^m - 2\pi^m a),$$

leading to:

$$a_n^S = \frac{\pi^m - s^m}{2\pi^m}(\in(a^W, \frac{1}{2})),$$

regardless of n*,* α *and* t*.*

O-B.1.3 Proof of Proposition 5

We first establish existence and uniqueness for t large enough (part 1), before studying the properties of symmetric equilibria (part 2).

Part 1. We first establish uniqueness, before turning to existence. Let $\mathcal{I} \equiv \{1, \ldots, n\}$ denote the set of platforms, $\mathbf{a} = (a_1, \ldots, a_n)$ the commissions set in stage $1a$. In the continuation equilibrium, in stage 2 platforms' consumer bases are characterized by (29). Hence, as $t \to +\infty$, each platform's consumer base satisfies $D_i \simeq D^\infty$ up to $O(1/t)$, where:

$$D^\infty = \frac{1}{n}.$$

Using (21) and (30), the platforms' app bases satisfy $y_{Ii} \simeq y_I^\infty(a_i)$ and $y_J \simeq y_J^\infty(\mathbf{a})$ up to $O(1/t)$, where:

$$\begin{aligned} y_I^\infty(a_i) &\equiv F_I((1-a_i)\frac{\pi^m}{n}), \\ y_J^\infty(\mathbf{a}) &\equiv F_J(\sum_{h\in\mathcal{I}}(1-a_h)\frac{\pi^m}{n}). \end{aligned}$$

$\mathcal{P}_i$'s subsidy therefore satisfies, up to $O(1/t)$:

$$\sigma_i \simeq \sigma_n^\infty(a_i, \mathbf{a}_{-i}) \equiv \alpha(s^m + a_i\pi^m)\, y_J^\infty(a_i, \mathbf{a}_{-i}) + (1-\alpha)\,\varphi(a_i),$$

where

$$\varphi(a) \equiv (s^m + a\pi^m)\, F_I\left(\frac{(1-a)\,\pi^m}{n}\right).$$

As noted in the text, in stage $1a$ each $\mathcal{P}_i$ seeks to maximize its subsidy advantage, equal to $\Delta_i = \sigma_i - \bar{\sigma}_i \simeq \Delta_n^\infty(a_i, \mathbf{a}_{-i})$, up to $O(1/t)$, where:

$$\Delta_n^\infty(a_i, \mathbf{a}_{-i}) \equiv (a_i - \bar{a}_i)\,\pi^m \alpha y_J^\infty(a_i, \mathbf{a}_{-i}) + (1-\alpha)\left[\varphi(a_i) - \frac{1}{n-1}\sum_{h\in\mathcal{I}\setminus\{i\}}\varphi(a_h)\right],$$

where

$$\bar{a}_i \equiv \frac{1}{n-1}\sum_{h\in\mathcal{I}\setminus\{i\}} a_h$$

denotes the average of $\mathcal{P}_i$'s rivals' commissions.

In what follows, we consider the limit game Γ_n^∞ in which each $\mathcal{P}_i$ sets a_i so as to maximize $\Delta_n^\infty(a_i, \mathbf{a}_{-i})$. We show that there exists a unique equilibrium, in which platforms' best-responses are moreover uniquely defined. By continuity, this establishes existence and uniqueness of the competitive equilibrium for t large enough.

We first note that, in any equilibrium of game Γ_n^∞, subsidies are non-negative:

Claim O-B.1 (non-negative subsidies) *In any equilibrium of game* Γ_n^∞, *all commissions are strictly higher than* a^W.

Proof. Consider a candidate equilibrium of game Γ^∞ yielding commissions $\mathbf{a} = (a_1, \ldots, a_n)$, app bases $y \equiv \alpha y_J^\infty(\mathbf{a})$ and $\{y_i \equiv (1-\alpha) y_I^\infty(a_i)\}_{i\in\mathcal{I}}$, and subsidies $\{\sigma_i \equiv (s^m + a_i\pi^m)(y + y_i)\}_{i\in\mathcal{I}}$. Without loss of generality, suppose that $a_i = \min_{h\in\mathcal{I}}\{a_h\}$, implying $y_i \geq y_h$ for any $h \neq i$, and let $\Delta_{ih} \equiv \sigma_i - \sigma_h$ denote $\mathcal{P}_i$'s subsidy advantage compared to a rival $\mathcal{P}_h$; finally, following a deviation to $a_i' \neq a_i$, let $y' \equiv y_J^\infty(a_i', \mathbf{a}_{-i})$, $y_i' \equiv y_I^\infty(a_i')$, $\sigma_i' \equiv (s^m + a_i'\pi^m)(y' + y_i')$, $\{\sigma_h' \equiv (s^m + a_h\pi^m)(y' + y_h')\}_{h\in\mathcal{I}\setminus\{i\}}$ and $\Delta_{ih}' \equiv \sigma_i' - \sigma_h'$ denote the resulting app bases, subsidies and subsidy advantages. We have:

- If $a_i < a^W$, then $\sigma_i < 0$ (as $s^m + a_i\pi^m < 0$ and $y + y_i \geq \alpha F_J\left(\left(1-a^W\right)\pi^m/n\right) + (1-\alpha) F_I\left(\left(1-a^W\right)\pi^m/n\right) > 0$); consider now a deviation to $a_i' = a^W$, implying $\sigma_i' = 0$ (as $s^m + a_i'\pi^m = 0$), $y' \leq y$ and $y_i' \leq y_i$. We have:
  - for every $j \neq i$ for which $a_j < a^W$, $\sigma_i \leq \sigma_j$ (as $s^m + a_i\pi^m \leq s^m + a_j\pi^m < 0$ and $y_i \geq y_j$) and $\sigma_j' < 0 = \sigma_i'$; hence, $\Delta_{ij} \leq 0 < -\sigma_j' = \Delta_{ij}'$, implying that the deviation strictly increases Δ_{ij};
  - for every $h \neq i$ for which $a_h = a^W$, $\sigma_h = \sigma_h' = 0$; hence, $\Delta_{ih} = \sigma_i - 0 < 0 = \Delta_{ih}'$, implying that the deviation strictly increases Δ_{ih} as well;
  - finally, for every $k \neq i$ for which $a_k > a^W$, $\sigma_k \geq \sigma_k' \geq 0$ (as $s^m + a_k\pi^m > 0$ and $y \geq y' \geq 0$); hence, $\Delta_i = \sigma_i - \sigma_k < \sigma_i' - \sigma_k' = \Delta_i'$ (as $\sigma_i < 0 = \sigma_i'$ and $\sigma_k \geq \sigma_k'$), implying that the deviation is once more strictly profitable.

  It follows that the deviation strictly increases $\Delta_i = (\sum_{h\in\mathcal{I}\setminus\{i\}} \Delta_{ih})/(n-1)$.

- If instead $a_i = a^W$, then $\sigma_i = 0$ (as $s^m + a_i\pi^m = 0$) and $\sigma_h \geq 0$, implying $\Delta_{ih} \leq 0$ for every $h \in \mathcal{I}\setminus\{i\}$ (as $s^m + a_h\pi^m \geq 0$ and $y + y_h \geq 0$); three cases can be distinguished:
  - If $\alpha = 0$ and $a_j = 1$ for every $j \neq i$, then any such j obtains $\Delta_j = 0$ (as $y = y_j = 0$) and could profitably deviate to, say, $a_j' = 0$, so as to obtain $\Delta_j' = \sigma_j' = s^m y_I^\infty(0) > 0 = \Delta_j$.
  - If instead $a_h = a^W$ for every $h \in \mathcal{I}$, then $\sigma_i = \sigma_h = \Delta_i = 0$ (as $s^m + a_i\pi^m = s^m + a_h\pi^m = 0$); a deviation to, say $a_i' = 0$ would lead to $\sigma_j' = 0$ (as $s^m + a_j\pi^m = 0$) for every $j \neq i$ and $\Delta_i' = \sigma_i' = s^m(y' + y_i') = s^m\left[\alpha y_J^\infty(0, \mathbf{a}_{-i}) + (1-\alpha) y_I^\infty(0)\right] > 0 = \Delta_i$, and would therefore be strictly profitable.

– In all other cases, $\sigma_j > 0$ for some $j \neq i$, and so $\Delta_i \leq \Delta_{ij}/(n-1) = -\sigma_j/(n-1) < 0$; a deviation to $a_i' = \bar{a}_i$ would lead to

$$\Delta_i' = (\bar{a}_i - \bar{a}_i)\,\pi^m y' + (1-\alpha)[\varphi(\bar{a}_i) - \frac{1}{n-1}\sum_{h\in\mathcal{I}\setminus\{i\}}\varphi(a_h)],$$

where

$$\begin{aligned}\varphi'(a) &\equiv \pi^m F_I\left(\frac{(1-a)\,\pi^m}{n}\right) - (s^m + a\pi^m)\,f_I\left(\frac{(1-a)\,\pi^m}{n}\right)\frac{\pi^m}{n},\\ \varphi''(a) &\equiv -2\pi^m f_I\left(\frac{(1-a)\,\pi^m}{n}\right)\frac{\pi^m}{n} + (s^m + a\pi^m)\,f_I'\left(\frac{(1-a)\,\pi^m}{n}\right)\left(\frac{\pi^m}{n}\right)^2.\end{aligned}$$

It follows from Assumption 3 that

$$\varphi''(a) < 0 \tag{O-B.3}$$

in the range $a \geq a^W$. Hence, from Jensen's inequality, $\Delta_i' \geq 0\,(> \Delta_i)$, implying that the deviation would be strictly profitable.

It follows that, in any equilibrium of game Γ_n^∞, alll commissions are strictly higher than a^W. ■

Any commission $a > 1$ discourages app development and is formally equivalent to $a' = 1$. Hence, it follows from Claim O-B.1 that, without loss of generality we can restrict attention to commissions lying in the range $(a^W, 1]$. Equilibrium commissions must therefore satisfy the first-order conditions, for $i \in \mathcal{I}$:

$$\begin{cases} d_n^\infty(a_i, \mathbf{a}_{-i}) \geq 0 & \text{if } a_i = 1, \\ d_n^\infty(a_i, \mathbf{a}_{-i}) = 0 & \text{otherwise,}\end{cases}$$

where:

$$\begin{aligned} d_n^\infty(a_i, \mathbf{a}_{-i}) &\equiv \frac{\partial \Delta_n^\infty(a_i, \mathbf{a}_{-i})}{\partial a_i} \\ &= \pi^m \alpha y_J^\infty(a_i, \mathbf{a}_{-i}) + (a_i - \bar{a}_i)\,\pi^m \alpha \frac{\partial y_J^\infty(a_i, \mathbf{a}_{-i})}{\partial a_i} + (1-\alpha)\varphi'(a_i)\,. \end{aligned}$$

Next, we show that any equilibrium must be symmetric:

Claim O-B.2 (symmetry) *In any equilibrium of game Γ_n^∞, commissions are symmetric.*

Proof. Let $a_i = \min_{h\in I}\{a_h\}$ and $a_j = \min_{h\in \mathcal{I}\setminus\{i\}}\{a_h\}$, which from the above analysis satisfy$a^W < a_i < a_j \leq 1$; we then have (using $\partial y_J^\infty/\partial a_i = \partial y_J^\infty/\partial a_j = -f_J(\cdot)\,\pi^m/n$ and $a_i - \bar{a}_i - (a_j - \bar{a}_j) = n\,(a_i - a_j)\,/\,(n-1)$):

$$d_n^\infty(a_i, \mathbf{a}_{-i}) - d_n^\infty(a_j, \mathbf{a}_{-j}) = \alpha\,(a_i - a_j)\,\frac{n\pi^m}{n-1}\frac{\partial y_J^\infty(a_i, \mathbf{a}_{-i})}{\partial a_i} + (1-\alpha)\left[\varphi'(a_i) - \varphi'(a_j)\right].$$

In the left-hand side of the above equation, both terms are weakly positive – as $a_i < a_j$ and $\partial y_J^\infty/\partial a_i < 0$, for the first term, and (from (O-B.3)) $\varphi'(a_i)$ is strictly decreasing, for the second term – and at least one of them is strictly positive – the first one if $\alpha > 0$, and the second one if $\alpha < 1$. Hence, $d_n^\infty(a_i, \mathbf{a}_{-i}) > d_n^\infty(a_j, \mathbf{a}_{-j})$, which in turn, implies that the commissions $(a_i, a_j, \mathbf{a}_{-i-j})$ cannot constitute an equilibrium: if $a_j < 1$, we should have $d_n^\infty(a_i, \mathbf{a}_{-i}) = d_n^\infty(a_j, \mathbf{a}_{-j}) = 0$, a contradiction; if instead $a_j = 1$, we should have $d_n^\infty(a_i, \mathbf{a}_{-i}) = 0 \leq d_n^\infty(a_j, \mathbf{a}_{-j})$, another contradiction. ■

Focusing on equilibria in which all platforms charge the same commission a (implying $y_J^\infty(\mathbf{a}) = \hat{y}_J^\infty(a) \equiv F_J((1-a)\,\pi^m)$); the first-order derivative then boils down to:

$$\hat{d}_n^\infty(a) \equiv \pi^m \alpha \hat{y}_J^\infty(a) + (1-\alpha)\varphi'(a),$$

which is strictly decreasing in a (as $d\hat{y}_J^\infty/da = -f_J((1-a)\,\pi^m)\,\pi^m < 0$ and $\varphi''(a) < 0$). For $\alpha = 1$, it reduces further to $F_J((1-a)\,\pi^m) = 0$, implying $a_n^C(\alpha) = 1$. For $\alpha < 1$, it satisfies

$$\begin{aligned}
\hat{d}_n^\infty(1) &= (1-\alpha)\,\varphi'(1) = -(1-\alpha)\,(s^m + a\pi^m)\,f_I(0)\,\frac{\pi^m}{n} < 0,\\
\hat{d}_n^\infty\left(a^W\right) &= \pi^m\left[\alpha F_J(s^m + \pi^m) + (1-\alpha)F_I\left(\frac{s^m + \pi^m}{n}\right)\right] > 0.
\end{aligned}$$

It follows that there is a unique candidate symmetric equilibrium, $a_n^C(\alpha)$, which moreover satisfies $a_n^C(1) = 1$ and $a_n^C(\alpha) \in \left(a^W, 1\right)$ otherwise.

To establish existence, we show that the function

$$\begin{aligned}
\psi(a) &\equiv \left.\Delta_n^\infty(a_i, \mathbf{a}_{-i})\right|_{a_i = a, a_j = a_n^C(\alpha) \text{ for } j \neq i}\\
&= \left(a - a_n^C(\alpha)\right)\pi^m \alpha F_J\left(\left[n - a - (n-1)\,a_n^C(\alpha)\right]\frac{\pi^m}{n}\right)\\
&\quad + (1-\alpha)\left[\varphi(a) - \varphi\left(a_n^C(\alpha)\right)\right]
\end{aligned}$$

is indeed maximal for $a = a_n^C(\alpha)$. We first note that:

$$\begin{aligned}\psi'(a) &= \pi^m \alpha F_J\left([n-a-(n-1)a_n^C(\alpha)]\frac{\pi^m}{n}\right) \\ &\quad - \left(a - a_n^C(\alpha)\right)\pi^m \alpha f_J\left([n-a-(n-1)a_n^C(\alpha)]\frac{\pi^m}{n}\right)\frac{\pi^m}{n} + (1-\alpha)\varphi'(a),\end{aligned}$$

which by construction satisfies $\psi'\left(a_n^C(\alpha)\right) = 0$, and

$$\begin{aligned}\psi''(a) &= -2\pi^m \alpha f_J\left([n-a-(n-1)a_n^C(\alpha)]\frac{\pi^m}{n}\right)\frac{\pi^m}{n} \\ &\quad + \left(a - a_n^C(\alpha)\right)\pi^m \alpha f_J'\left([n-a-(n-1)a_n^C(\alpha)]\frac{\pi^m}{n}\right)\left(\frac{\pi^m}{n}\right)^2 + (1-\alpha)\varphi''(a)\end{aligned}$$

For $a \geq a_n^C(\alpha)$, $\psi''(a) < 0$ as $f_J(\cdot) > 0 \geq f_J'(\cdot)$, $\varphi''(a) < 0$ and $\alpha \in [0,1]$. Hence, in the range $a \geq a_n^C(\alpha)$, $\psi(a)$ is maximal for $a = a_n^C(\alpha)$.

Furthermore, for $a \leq a_n^C(\alpha)$, we have $\psi(a) \leq \hat{\psi}(a)$, where (replacing a with $a_n^C(\alpha)$ in the expression of $y_J^\infty(\cdot)$):

$$\hat{\psi}(a) \equiv \left[a - a_n^C(\alpha)\right]\pi^m \alpha F_J\left(\left[1 - a_n^C(\alpha)\right]\pi^m\right) + (1-\alpha)\left[\varphi(a) - \varphi\left(a_n^C(\alpha)\right)\right].$$

By construction, $\hat{\psi}\left(a_n^C(\alpha)\right) = \psi\left(a_n^C(\alpha)\right) = 0$ and $\hat{\psi}'\left(a_n^C(\alpha)\right) = \psi'\left(a_n^C(\alpha)\right) = 0$. Furthermore:

$$\hat{\psi}''(a) = (1-\alpha)\varphi''(a) \leq 0$$

where the inequality stems from $\varphi''(a) < 0$ and $\alpha \leq 1$. It follows that, in the range $a \leq a_n^C(\alpha)$, $\psi(a)$ is again maximal for $a = a_n^C(\alpha)$.

Part 2. We now focus on symmetric candidate equilibria, in which:

- all platforms set the same commission a and charge the same price p to consumers;
- following a deviation by one platform, all other platforms keep charging a symmetric price, and thus share evenly their aggregate consumer base.

By construction, in equilibrium each platform obtains a share $1/n$ of consumers and an app base equal to $\hat{y}_n^S(a)$, given by (O-B.2). Consider now a deviation by $\mathcal{P}_1$ to some $a_1 \neq a$, say, and let D_1 denote the resulting consumer base for that platform. The app base of each $\mathcal{P}_i$ is then given by $y_i = \alpha y_J + (1-\alpha)y_{Ii}$, where (using $a_i = a$

and $D_i = (1 - D_1) / (n-1)$ for $i = 2, ..., n$):

$$\begin{aligned} y_{I1} &= F_I\left((1-a_1)\,\pi^m D_1\right), \\ y_{I2} &= \cdots = y_{In} = F_I\left((1-a)\,\pi^m \frac{1-D_1}{n-1}\right), \\ y_J &= F_J\left((1-a)\,\pi^m + (a-a_1)\,\pi^m D_1\right). \end{aligned}$$

Furthermore, from (29):

$$D_1 \equiv \frac{1}{n} + \frac{n-1}{2n-1}\frac{\Delta_1}{nt},$$

where (using $y_{Ii} = y_{In}$ for $i = 2, ..., n-1$):

$$\begin{aligned} \Delta_1 &= (s^m + a_1\pi^m)\left[\alpha y_J + (1-\alpha)\,y_{I1}\right] - (s^m + a\pi^m)\left[\alpha y_J + (1-\alpha)\,y_{In}\right] \\ &= (1-\alpha)\left[(s^m + a_1\pi^m)\,y_{I1} - (s^m + a\pi^m)\,y_{In}\right] + \alpha\,(a_1 - a)\,\pi^m y_J. \end{aligned}$$

Differentiating leads to:

$$\frac{dy_{I1}}{da_1} = f_I\left((1-a_1)\pi^m D_1\right)\pi^m[(1-a_1)\frac{dD_1}{da_1} - D_1], \tag{O-B.4}$$

$$\frac{dy_{In}}{da_1} = -f_I\left((1-a)\,\pi^m\frac{1-D_1}{n-1}\right)(1-a)\,\frac{\pi^m}{n-1}\frac{dD_1}{da_1}, \tag{O-B.5}$$

$$\frac{dD_1}{da_1} = \frac{n-1}{(2n-1)\,nt}\frac{d\Delta_1}{da_1}, \tag{O-B.6}$$

and (using $D_1 = 1/n$ and $y_1 = \hat{y}_n^S(a)$ for $a_1 = a$):

$$\left.\frac{d\Delta_1}{da_1}\right|_{a_1=a} = \hat{y}_n^S(a)\,\pi^m + (1-\alpha)\,(s^m + a\pi^m)\left[\left.\frac{dy_{I1}}{da_1}\right|_{a_1=a} - \left.\frac{dy_{In}}{da_1}\right|_{a_1=a}\right]. \tag{O-B.7}$$

If $\alpha = 1$, (O-B.7) boils down to:

$$\left.\frac{d\Delta_1}{da_1}\right|_{a_1=a} = \hat{y}^S(a)\,\pi^m \geq 0,$$

with a strict inequality for $a < 1$. It follows that the only equilibrium is $a = 1$.

If $\alpha < 1$, it follows from (O-B.2), (O-B.4), (O-B.5) and (O-B.7) that (using $D_1 = 1/n$ for $a_1 = a$):

$$\left.\frac{d\Delta_1}{da_1}\right|_{a_1=a=1} = -\,(1-\alpha)\,(s^m + a\pi^m)\,f_I(0)\,\frac{\pi^m}{n} < 0,$$

implying that, starting from a candidate equilibrium in which $a = 1$, each platform would have an incentive to lower its commission. Hence, in equilibrium, $a < 1$. We must therefore have

$$\left.\frac{d\Delta_1}{da_1}\right|_{a_1=a} = 0.$$

It then follows from (O-B.4) – (O-B.6) that:

$$\begin{aligned}\left.\frac{dy_{In}}{da_1}\right|_{a_1=a} &= \left.\frac{dD_1}{da_1}\right|_{a_1=a} = 0,\\ \left.\frac{dy_{I1}}{da_1}\right|_{a_1=a} &= -f_I\left(\frac{(1-a)\,\pi^m}{n}\right)\frac{\pi^m}{n}.\end{aligned}$$

Plugging in these expressions and (O-B.2) into (O-B.7), and dividing by $(1-\alpha)\,\pi^m$, then yields (where the superscript S refers to Spokes):

$$0 = \phi_C^S\,(a;\alpha,n) \equiv \frac{\alpha}{1-\alpha}F_J\,((1-a)\pi^m) + F_I\left(\frac{(1-a)\pi^m}{n}\right) - \frac{s^m+a\pi^m}{n}f_I\left(\frac{(1-a)\pi^m}{n}\right),$$

where $\phi_C^S\,(\cdot)$ satisfies:

$$\begin{aligned}\phi_C^S\left(a^W;\alpha\right) &= \frac{\alpha}{1-\alpha}F_J\,(s^m+\pi^m) + F_I\left(\frac{s^m+\pi^m}{n}\right) > 0,\\ \phi_C^S\,(1;\alpha) &= -\frac{s^m+\pi^m}{n}f_I\,(0) < 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\phi_C^S}{\partial a}\,(a;\alpha,n) \equiv -\frac{\alpha}{1-\alpha}f_J\,((1-a)\pi^m)\,\pi^m - 2f_I\left(\frac{(1-a)\pi^m}{n}\right)\frac{\pi^m}{n}\\ + \frac{s^m+a\pi^m}{n}f_I'\left(\frac{(1-a)\pi^m}{n}\right)\frac{\pi^m}{n} < 0,\end{aligned}$$

where the inequalities stem from Assumption 3. Furthermore, $\phi_C^S\,(a;\alpha,n)$ is strictly increasing in α and, for any $a<1$, tends to infinity as α tends to 1. It follows that there is a unique symmetric equilibrium, $a = a_n^C\,(\alpha)$, where $a_n^C\,(\alpha) \in \left(a^W,1\right)$, is increasing in α, and tends to 1 as α does so. Finally, as n goes to infinity, we have:

$$\lim_{n\longrightarrow+\infty}\phi_C^S\,(a;\alpha,n) \equiv \frac{\alpha}{1-\alpha}F_J\,((1-a)\pi^m)\,,$$

implying that $a_n^C\,(\alpha) \in \left(a^W,1\right)$ tends again to 1 as n goes to infinity.

To conclude the proof, we now compare $a_n^C\,(\alpha)$ to $a_n^S\,(\alpha)$. For $\alpha = 1$, we have

$a_n^S(\alpha) < 1 = a_n^C(\alpha)$. For $\alpha < 1$, $a_n^S(\alpha)$ is characterized by the optimality condition:

$$\begin{aligned} 0 &= \frac{d\hat{\sigma}_n^S}{da}(a) = \pi^m \hat{y}_n^S(a) + (s^m + a\pi^m)\frac{d\hat{y}_n^S}{da}(a) \\ &= \pi^m \left[\alpha F_J((1-a)\pi^m) + (1-\alpha) F_I\left(\frac{(1-a)\pi^m}{n}\right)\right] \\ &\quad - (s^m + a\pi^m)\left[\alpha f_J((1-a)\pi^m)\pi^m + (1-\alpha) f_I\left(\frac{(1-a)\pi^m}{n}\right)\frac{\pi^m}{n}\right] \\ &= (1-\alpha)\pi^m \left\{\phi_C^S(a;\alpha,n) - \frac{\alpha}{1-\alpha}(s^m + a\pi^m) f_J((1-a)\pi^m)\right\}. \end{aligned}$$

It follows that

$$\phi_C^S\left(a_n^S(\alpha);\alpha,n\right) = \frac{\alpha}{1-\alpha}\left(s^m + a_n^S(\alpha)\pi^m\right) f_J\left((1-a_n^S(\alpha))\pi^m\right) > 0,$$

where the inequality stems from $a_n^S(\alpha) > a^W$. Together with the fact that $\phi_C^S(a;\alpha,n)$ is decreasing in a, this in turn implies that $a_n^S(\alpha) < a_n^C(\alpha)$.

O-B.2 Sequential Development

Let r_i denote the revenue offered by $\mathcal{P}_i$ to successful apps and $\rho_i \equiv [\lambda + (1-\lambda)\eta]\, r_i$ the resulting expected app revenue. In equilibrium, we have $a_1 = a_2 = a^C$, $r_1 = r_2 = r^C = \left(1 - a^C\right)\pi^m/2$ and $\rho_1 = \rho_2 = \rho^C = [\lambda + (1-\lambda)\eta]\, r^C$, and development and porting decisions are as illustrated by Figure 2.

To assess the impact of a platform's commission on its rival's app base, suppose that, starting from the equilibrium commissions, $\mathcal{P}_1$ slightly deviates and raises its commission by $da_1 > 0$. By construction, $a_1 = a^C$ maximizes $\mathcal{P}_1$'s profit, given $a_2 = a^C$; in the Hotelling setting, this means that $a_1 = a^C$ maximizes $\mathcal{P}_1$'s market share on the consumer side; it follows that the deviation has only a second-order effect on platforms' consumer bases. We can thus focus on the impact of da_1 on the platforms' app base through its direct impact on the revenues offered by the platforms – namely, r_1 is reduced by $dr_1 = \pi^m da_1/2$, whereas r_2 remains at its equilibrium value, $r^C \equiv a^C\pi^m/2$.

The numbers of apps in the first and second groups are respectively

$$y^S \equiv F\left(\rho^C\right)\left[1 - F\left(\hat{r}^C\right)\right] \text{ and } y^M \equiv \int_{\underline{k}}^{\hat{\rho}^C} F(k)\, dF(k) + \int_{\hat{\rho}^C}^{\hat{r}^C} F\left(\phi^C(k)\right) dF(k),$$

where

$$\hat{\rho}^C \equiv \frac{\rho^C + \lambda r^C}{1 + \lambda\delta} \text{ and } \hat{r}^C \equiv \frac{r^C}{\delta}.$$

The total number of apps available on each platform, weighted by their popularity, is then equal to (noting that a proportion λ of the y^M potentially multihoming apps developed on the rival platform are ported):

$$y^C \equiv [\lambda + (1-\lambda)\eta] \left(y^S + y^M\right) + \lambda y^M.$$

Suppose now that $\mathcal{P}_1$ raises its commission by $da_1 > 0$, thus *reducing* r_1 by $dr = da_1 \pi^m/2$ and ρ_1 by $d\rho = [\lambda + (1-\lambda)\eta]\, dr$, leaving r_2 and ρ_2 unchanged: $r_2 = r^C$ and $\rho_2 = \rho^C$. This induces five changes in the app bases, as illustrated by Figure 5:

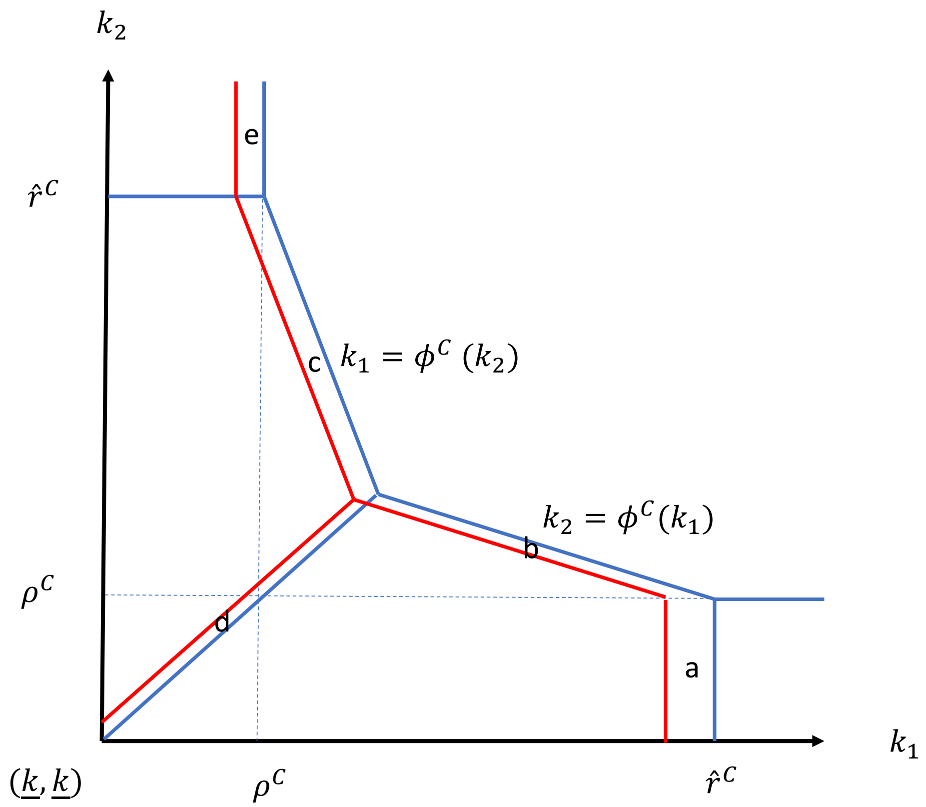

Figure 5: Impact of platform 1's commission on development and porting decisions.

a. developers with
$$k_1 \in \left(\hat{r}^C - \frac{dr}{\delta}, \hat{r}^C\right) \text{ and } k_2 < \rho^C,$$
initially developing their apps for $\mathcal{P}_2$ and porting it in case of success, keep developing their apps but no longer port them;

b. developers with
$$k_1 \in \left(\hat{\rho}^C, \hat{r}^C\right) \text{ and } k_2 \in \left(\phi^C(k_1) - \lambda dr, \phi^C(k_1)\right),$$
initially developing their apps for $\mathcal{P}_2$ and porting it in case of success, drop out;

c. developers with

$$k_2 \in \left(\hat{\rho}^C, \hat{r}^C\right) \text{ and } k_1 \in \left(\phi^C(k_2) - d\rho, \phi^C(k_2)\right),$$

initially developing their apps for $\mathcal{P}_1$ and porting it in case of success, drop out as well;

d. developers with[52]

$$k_1 \in \left(\underline{k}, \hat{\rho}^C\right) \text{ and } k_2 \in \left(k_1, k_1 + \frac{(1-\lambda)\eta}{1-\lambda\delta} dr\right),$$

switch from $\mathcal{P}_1$ to $\mathcal{P}_2$ as development platform (and keep porting their apps in case of success);

e. finally, developers with

$$k_2 \leq \left(\hat{r}^C, \bar{k}\right) \text{ and } k_1 \in \left(\rho^C - d\rho, \rho^C\right),$$

initially developing their apps solely for $\mathcal{P}_1$, drop out.

The changes mentioned in a and e have no impact on $\mathcal{P}_2$'s app base: developers in a keep developing their apps for $\mathcal{P}_2$, and those in e are never present on $\mathcal{P}_2$ anyway. Any change mentioned in b reduces instead $\mathcal{P}_2$'s app base by a factor $\lambda + (1-\lambda)\eta$, and any change listed in c reduces it by a factor λ (as these apps are ported onto $\mathcal{P}_2$ only when successful). By contrast, any change mentioned in d increases $\mathcal{P}_2$'s app base by a factor $[\lambda + (1-\lambda)\eta] - \lambda = (1-\lambda)\eta$ (as these apps are now first developed on $\mathcal{P}_2$, rather than being ported on $\mathcal{P}_2$ if successful). The overall impact on $\mathcal{P}_2$'s app base is equal to:

$$\begin{aligned} dy_2 &= -[\lambda + (1-\lambda)\eta] \int_{\hat{\rho}^C}^{\hat{r}^C} f\left(\phi^C(k_1)\right) \lambda dr dF(k_1) - \lambda \int_{\hat{\rho}^C}^{\hat{r}^C} f\left(\phi^C(k_2)\right) d\rho dF(k_2) \\ &\quad + (1-\lambda)\eta \int_{\underline{k}}^{\hat{\rho}^C} f(k_1) \frac{(1-\lambda)\eta}{1-\lambda\delta} dr dF(k_1). \end{aligned}$$

It follows that raising $\mathcal{P}_1$'s commission reduces the weighted number of apps available

[52]Conditional on porting the app if successful, a developer is indifferent between developing first for one or the other platform if

$$\rho_1 + \lambda r_2 - k_1 - \lambda\delta k_2 = \rho_2 + \lambda r_1 - k_2 - \lambda\delta k_1,$$

which amounts to $k_2 - k_1 = (d\rho - \lambda dr)/(1-\lambda\delta) = (1-\lambda)\eta/(1-\lambda\delta)$.

on $\mathcal{P}_2$ if and only if:

$$(1-\lambda\delta)\,2\lambda\frac{\lambda+(1-\lambda)\eta}{(1-\lambda)^2\,\eta^2} > \frac{\int_{\underline{k}}^{\hat{\rho}^C} f^2(k)\,dk}{\int_{\hat{\rho}^C}^{\hat{r}^C} f\left(\phi^C(k)\right) f(k)\,dk},$$

where the left-hand side is decreasing in η and tends to infinity as η tends to 0. It follows that the condition holds whenever the revenue generated by unsuccessful apps is small enough (i.e., η low enough).[53]

[53]This assumes that app development is not choked off (i.e., $\hat{\rho}^C = [2\lambda + (1-\lambda)\,\eta]\, r^C / (1+\lambda\delta) > (\underline{k} >)\, 0$), implying $r^C > 0$ and

$$\hat{r}^C - \hat{\rho}^C = \frac{r^C}{\delta} - \frac{2\lambda + (1-\lambda)\,\eta}{1+\lambda\delta} r^C = \frac{1-[\lambda+(1-\lambda)\,\eta]\,\delta}{\delta\,(1+\lambda\delta)} r^C > 0.$$

If app development is chocked off, then trivially $a^C > a^S$.